Unit 3 Planning the Unit

In this unit students analyze two- and three- dimensional figures exploring angle relationships, similarity, transformations, distance, area and volume. They understand and apply the Pythagorean Theorem and its converse to find distance on the coordinate plane, length, and investigate problem situations.

Vocabulary Development

The key terms for this unit can be found on the Unit Opener page. These terms are divided into Academic Vocabulary and Math Terms. Academic Vocabulary includes terms that have additional meaning outside of math. These terms are listed separately to help students transition from their current understanding of a term to its meaning as a mathematics term. To help students learn new vocabulary:

- Have students discuss meaning and use graphic organizers to record their understanding of new words.
- Remind students to place their graphic organizers in their math notebooks and revisit their notes as their understanding of vocabulary grows.
- As needed, pronounce new words and place pronunciation guides and definitions on the class Word Wall.

Embedded Assessments

Embedded Assessments allow students to do the following:

- Demonstrate their understanding of new concepts.
- Integrate previous and new knowledge by solving real-world problems presented in new settings.

They also provide formative information to help you adjust instruction to meet your students' learning needs.

Prior to beginning instruction, have students unpack the first Embedded Assessment in the unit to identify the skills and knowledge necessary for successful completion of that assessment. Help students create a visual display of the unpacked assessment and post it in your class. As students learn new knowledge and skills, remind them that they will be expected to apply that knowledge to the assessment. After students complete each Embedded Assessment, turn to the next one in the unit and repeat the process of unpacking that assessment with students.

CollegeBoard

Algebra / AP / College Readiness

This unit develops students' concept of geometric concepts by:

- Creating and using representations to analyze relationships and solve problems.
- Modeling written descriptions of physical situations.
- Explaining and justifying mathematical conclusions verbally and in writing using accurate and precise language.

Unpacking the Embedded Assessments

The following are the key skills and knowledge students will need to know for each assessment.

Embedded Assessment 1

Angle Measures, *Light and Glass*

- Complementary and supplementary angles
- Angles of a triangle or quadrilateral
- Angles formed by parallel lines cut by a transversal

Embedded Assessment 2

Rigid Transformations, *In Transformations We Trust*

- Translations, reflections, and rotations
- Transformations that preserve congruence

Embedded Assessment 3

Similarity and Dilations, *Business As Usual*

- Similar figures
- Dilations

Embedded Assessment 4

The Pythagorean Theorem, *Camp Euclid*

- Apply the Pythagorean Theorem
- Converse of the Pythagorean Theorem

Embedded Assessment 5

Surface Area and Volume, *Air Dancing*

- Surface area and lateral area of solids
- Volume of solids and composite solids

Suggested Pacing

The following table provides suggestions for pacing using a 45-minute class period. Space is left for you to write your own pacing guidelines based on your experiences in using the materials.

	45-Minute Period	Your Comments on Pacing
Unit Overview/Getting Ready	1	
Activity 16	4	
Activity 17	3	
Embedded Assessment 1	1	
Activity 18	4	
Activity 19	4	
Embedded Assessment 2	1	
Activity 20	4	
Activity 21	4	
Embedded Assessment 3	1	
Activity 22	3	
Activity 23	3	
Activity 24	2	
Embedded Assessment 4	1	
Activity 25	2	
Activity 26	4	
Embedded Assessment 5	1	
Total 45-Minute Periods	**43**	

Additional Resources

Additional resources that you may find helpful for your instruction include the following, which may be found in the eBook Teacher Resources.

- Unit Practice (additional problems for each activity)
- Getting Ready Practice (additional lessons and practice problems for the prerequisite skills)
- Mini-Lessons (instructional support for concepts related to lesson content)

Geometry

3

Unit Overview

In this unit you will continue your study of angles and triangles and explore the Pythagorean Theorem. You will investigate 2- and 3-dimensional figures and apply formulas to determine the area and volume of those figures. You will explore rigid transformations of figures, including translations, rotations, and reflections of two-dimensional figures.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- alternate
- transform

Math Terms

- angle
- ray
- complementary angles
- supplementary angles
- congruent
- transversal
- alternate exterior angles
- alternate interior angles
- corresponding angles
- vertical angles
- exterior angle of a triangle
- remote interior angle
- diagonal
- transformation
- preimage
- image
- translation
- reflection
- line of reflection
- equidistant
- rotation
- center of rotation
- composition of transformations
- similar figures
- similarity statement
- proportion
- scale factor
- dilation
- center of dilation
- scale factor of dilation
- hypotenuse
- legs
- Pythagorean Theorem
- surface area
- lateral area

ESSENTIAL QUESTIONS

? What are transformations and how are they useful in solving real-world problems?

? How are two- and three-dimensional figures related?

EMBEDDED ASSESSMENTS

These assessments, following activities 17, 19, 21, 24, and 26, will give you an opportunity to demonstrate how you can use your understanding of angles, triangles, transformation, and geometric formulas to solve problems.

Embedded Assessment 1:	
Angle Measures	p. 229
Embedded Assessment 2:	
Rigid Transformations	p. 263
Embedded Assessment 3:	
Similarity and Dilations	p. 293
Embedded Assessment 4:	
The Pythagorean Theorem	p. 325
Embedded Assessment 5:	
Surface Area and Volume	p. 353

Developing Math Language

As this unit progresses, help students make the transition from general words they may already know (the Academic Vocabulary) to the meanings of those words in mathematics. You may want students to work in pairs or small groups to facilitate discussion and to build confidence and fluency as they internalize new language. Ask students to discuss new academic and mathematics terms as they are introduced, identifying meaning as well as pronunciation and common usage. Remind students to use their math notebooks to record their understanding of new terms and concepts.

As needed, pronounce new terms clearly and monitor students' use of words in their discussions to ensure that they are using terms correctly. Encourage students to practice fluency with new words as they gain greater understanding of mathematical and other terms.

Unit Overview

Ask students to read the unit overview and mark the text to identify key phrases that indicate what they will learn in this unit.

Materials

- blackline masters
- calculators
- graph paper
- index cards
- masking tape
- paper clips
- pictures/models of rectangular prisms and pyramids
- protractor
- rulers
- scissors
- shoebox
- small mirrors
- sticky notes
- tape measures
- timer

Key Terms

As students encounter new terms in this unit, help them to choose an appropriate note taking technique such as a graphic organizer for their word study. Encourage students to make notes to help them remember the meaning of new words. Refer students to the Glossary to review translations of key terms as needed. Have students place their notes in their math notebooks and revisit as needed as they gain additional knowledge about each word or concept.

Essential Questions

Read the essential questions with students and ask them to share possible answers. As students complete the unit, revisit the essential questions to help them adjust their initial answers as needed.

Unpacking Embedded Assessments

Prior to beginning the first activity in this unit, turn to Embedded Assessment 1 and have students unpack the assessment by identifying the skills and knowledge they will need to complete the assessments successfully. Guide students through a close reading of the assessment, and use a graphic organizer or other means to capture their identification of the skills and knowledge. Repeat the process for each Embedded Assessment in the unit.

UNIT 3
Getting Ready

Use some or all of these exercises for formative evaluation of students' readiness for Unit 3 topics.

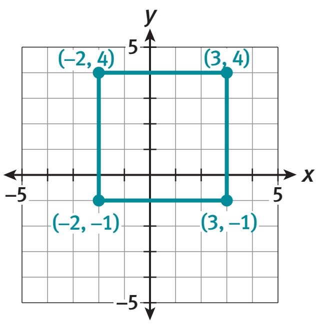
Prerequisite Skills

- Coordinate Plane (Items 1, 2) 6.NS.C.6c
- Triangles (Item 3) 4.G.A.2
- Ratio and Proportion (Items 4, 5) 6.RP.A.3, 7.RP.A.3
- Perimeter and Area (Items 6, 7, 8) 6.G.A.1, 7.G.B.4, 7.G.B.6

Answer Key

1. $A(3, 7)$; $B(-1, 5)$; $C(6, 0)$; $D(-5, -4)$

2.



3. **a.** An acute angle is an angle with a measure less than 90°.

b. A right angle is an angle that measures exactly 90°.

c. An obtuse angle is an angle whose measure is greater than 90°and less than 180°.

4. Answers may vary. $\frac{6}{15}, \frac{8}{20}, \frac{10}{25}$

5. **a.** $x = 9$

b. $x = 12$

6. **a.** $P = 16$ units

b. $P = 42$ units

c. $C = 6\pi \approx 18.85$ units

d. $P = 28$ units

7. **a.** $A = 14.31$ square units

b. $A = 84$ square units

c. $A = 9\pi \approx 28.27$ square units

d. $A = 36$ square units

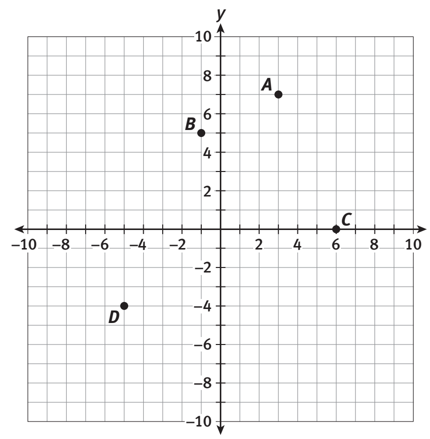
8. $A_{shaded} = \frac{1}{2}bh - \pi r^2$; Explanations may vary.

UNIT 3

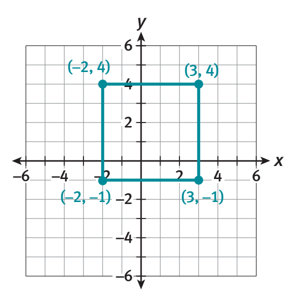
Getting Ready

Write your answers on notebook paper. Show your work.

1. Give the coordinates of points A, B, C, and D on the graph below.



2. On the grid below, draw a square that has $(-2, 4)$ and $(3, -1)$ as two of its vertices. Label the other two vertices.



3. Define the following terms:

a. acute triangle

b. right triangle

c. obtuse triangle

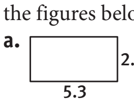
4. Write three ratios equivalent to $\frac{2}{5}$.

5. Find the value of x in each of the following.

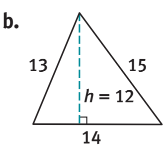
a. $\frac{2}{3} = \frac{6}{x}$ **b.** $\frac{4}{7} = \frac{x}{21}$

6. Find the perimeter or circumference of each of the figures below.

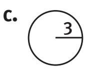
a.



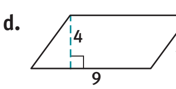
b.



c.

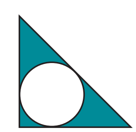


d.



7. Find the area of each figure in Item 6.

8. Explain using specific formulas how you could find the area of the shaded area of the figure below.



Getting Ready Practice

For students who may need additional instruction on one or more of the prerequisite skills for this unit, Getting Ready practice pages are available in the eBook Teacher Resources. These practice pages include worked-out examples as well as multiple opportunities for students to apply concepts learned.

Angle-Pair Relationships

ACTIVITY 16

The Winning Angle

Lesson 16-1 Complementary and Supplementary Angles

My Notes

Learning Targets:

- Identify and determine the measure of complementary angles.
- Identify and determine the measure of supplementary angles.

SUGGESTED LEARNING STRATEGIES: Visualization, Create Representations, Graphic Organizer, Identify a Subtask

Bob Toose, football coach and geometry teacher at Johnny Unitas High School, names his football plays after different geometric terms. He knows that the players from other schools won't know what is coming at them with these names.

Coach Toose gives playbook quizzes to make sure his players know their plays. A portion of one of his quizzes is below.

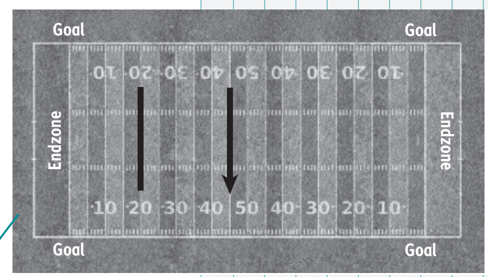
1. Match each play shown at the right with the mathematical term that best describes it. Draw a line to connect the plays and the terms.

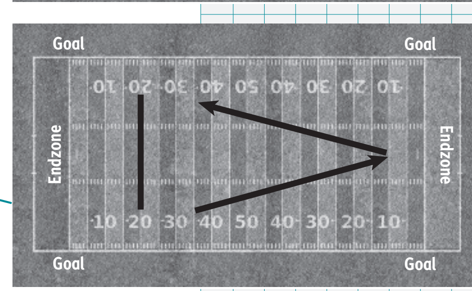
Angle

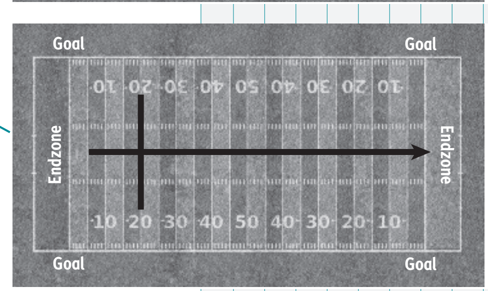
Perpendicular Lines

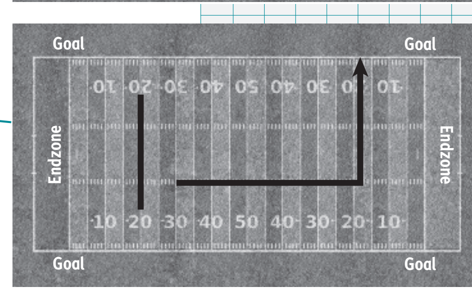
Parallel Lines

Right Angle









Common Core State Standards for Activity 16

8.G.A.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

ACTIVITY 16

Guided

Activity Standards Focus

In this activity, students establish facts about angle pairs, including complementary, supplementary, and vertical angles. Students also investigate and apply the relationships among angle pairs when parallel lines are cut by a transversal.

Lesson 16-1

PLAN

Materials

- protractor

Pacing: 1–2 class periods

Chunking the Lesson

#1	#2–5
Example A	#6–10

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Give students 3 or 4 minutes to write down what they know about angles. Then have students share their work within their groups. Debriefing in this way will activate students' prior knowledge about angles and serve as a natural segue into the content of this lesson.

1 Activating Prior Knowledge, Visualization, Think-Pair-Share, Sharing and Responding, Interactive Word Wall Students need to match the play diagrams on this page with the mathematical terms for each of the play diagrams. The concepts shown in these diagrams are concepts that students have studied in previous math courses. Sharing and Responding allows students who have not yet mastered the material to hear the names for these diagrams. These can be posted in the classroom on the Interactive Word Wall.

Teacher to Teacher

This activity will allow some of the football fans in your classroom to share their knowledge of the game with other students. Allow them to explain what the plays in the diagrams look like on the field.

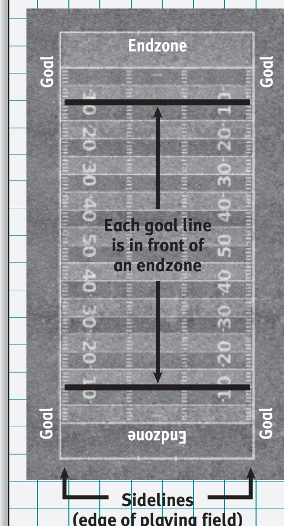
2–5 Visualization, Create Representations, Graphic Organizer, Marking the Text, Interactive Word Wall Students have studied angles in previous math courses. This is an opportunity to informally assess understanding of angle measures. Be sure to bring out the idea that the term *complementary* applies to pairs of angles only. Some students may believe that the term can apply to more than two angles. Students may have an easier time coming up with complementary angle measures than illustrating such angles.

If students need more instruction on protractor use, take a timeout from the lesson and have students measure angles of different sizes. See the mini-lesson below.

Developing Math Language

This lesson introduces the terms *complementary* and *supplementary*. As needed, pronounce these new terms clearly and monitor students' pronunciation of the terms in their class discussions. Use the class word wall to keep new terms in front of students. Include pronunciation guides as needed. Encourage students to review the word wall regularly and to monitor their own understanding and use of new terms in their group discussions.

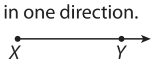
My Notes



MATH TERMS

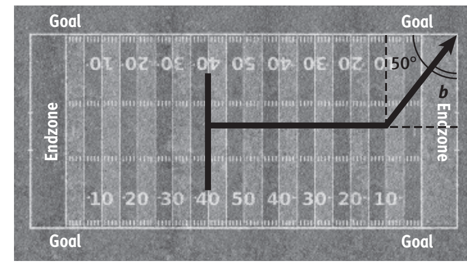
An **angle** is the union of two rays with a common endpoint called the vertex.

A ray is part of a line with one endpoint. It extends indefinitely in one direction.



Ray *XY* can be written $\overrightarrow{XY}$. Its endpoint is *X* and it extends forever through point *Y*.

Coach Toose is very particular about the routes that his players run. He told his receiver that this "corner" route needed to be run at a 50° angle to the sideline of the end zone.

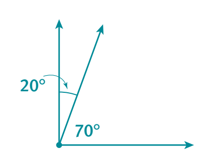


2. What is the measure of angle *b* in the diagram above?
40°

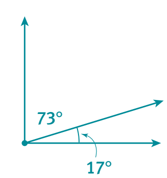
3. Two angles are ***complementary*** if the sum of their measures is 90°. Explain why these two angles can be classified as complementary.
The angles are complementary because 50° + 40° = 90°.

4. Coach Toose wanted his players to run other corner routes as well. Identify the ***angle*** complementary to the one listed. Draw a diagram to illustrate the angle and its complement.

a. 20°
70°



b. 73°
17°



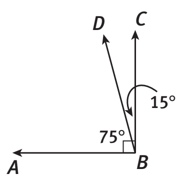
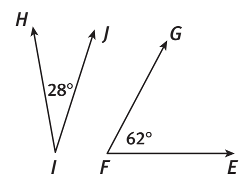
MINI-LESSON: Measuring Angles with a Protractor

If students need additional support in measuring angles, a mini-lesson is available to provide practice. The mini-lesson includes step-by-step instructions on how to measure an angle with a protractor.

See SpringBoard's eBook Teacher Resources for a student page for this mini-lesson.

Lesson 16-1
Complementary and Supplementary Angles

5. **Make use of structure.** Shown below is an example of two pairs of angles. Compare and contrast the angle pairs.

Angle Pair #1	Angle Pair #2
	

Sample answer: Both pairs of angles are complementary, but one pair of angles are adjacent and the other pair of angles are not adjacent.

Make notes about the math terms and academic vocabulary used in Example A below and other examples that follow. Review your notes and use new vocabulary when you write and discuss your responses to items.

Example A

The measure of angle A is $(3x)°$ and the measure of its complement, angle B, is $(x + 6)°$. Determine the measures of the angles.

Step 1: Write an equation that shows the relationship between $\angle A$ and $\angle B$.

The sum of the angle measures is 90°. $m\angle A + m\angle B = 90°$.

Substitute the expressions for the angle measures. $3x + x + 6 = 90$

Step 2: Solve the equation.

Original equation	$3x + x + 6 = 90$
Combine like terms.	$4x + 6 = 90$
Subtract 6 from both sides.	$4x + 6 - 6 = 90 - 6$
Simplify.	$4x = 84$
Divide both sides by 4.	$\frac{4x}{4} = \frac{84}{4}$
Simplify.	$x = 21$

Step 3: Determine the measure of the two angles.

$$m\angle A = (3x)° = (3 \cdot 21)° = 63°$$
$$m\angle B = (x + 6)° = (21 + 6)° = 27°$$

Solution: $m\angle A = 63°$, and $m\angle B = 27°$.

Try These A

Angle P and angle Q are complementary. Determine the measures of the angles.

a. $m\angle P = (2x - 5)°$ and $m\angle Q = (x + 20)°$ **45°; 45°**

b. $m\angle P = (x + 4)°$ and $m\angle Q = (5x - 4)°$ **19°; 71°**

My Notes

READING MATH

$m\angle A$ is read "the measure of angle A."

Example A Create Representations, Identify a Subtask This example takes students to a more abstract level. Be sure students understand that the measure of the angle is not the same as the value of x. Students must substitute their value for x into the given expression for each angle's measure to determine their answers. Be sure students realize that they can check their answers for the angle measures by calculating the sum and making sure it is 90°.

6–10 Visualization, Create Representations, Sharing and Responding, Discussion Groups, Interactive Word Wall Have students share their sketches of supplementary angles from Item 6 with their partners and their groups so that they can compare the appearance of various pairs of supplementary angles. Item 8 is similar to Item 4 but is done with supplementary angles rather than complementary angles. Students may question how to draw an angle that measures 153.1°. Explain that the drawing is an approximation and does not need to be exact, so using a measure very close 153° is appropriate. In Item 8, students may all draw supplementary angles that are adjacent. Item 9 asks students to create a representation of supplementary angles that are not adjacent. It is important for students to understand that a pair of angles can be supplementary without being adjacent. Class discussion after these items will ensure that everyone understands what it means for angles to be supplementary.

Teacher to Teacher

To help students remember *complementary* and *supplementary* and which is 90° and 180°, remind them that *c* for complementary comes before *s* for supplementary, just as 90° comes before 180°.

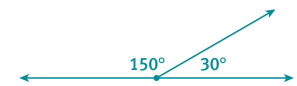
Developing Math Language

As you guide students through their learning of these essential new mathematical terms, explain meanings in ways that are accessible for your students. As much as possible, provide concrete examples to help students gain understanding. Encourage students to make notes about new terms and their understanding of what they mean and how to use them to describe precise mathematical concepts and processes.

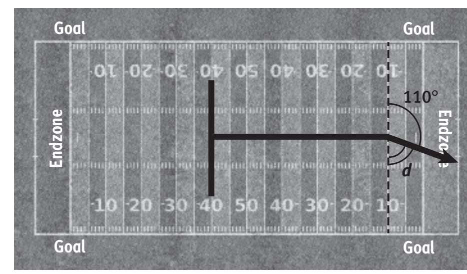
My Notes

Another route that Coach Toose has his players run is a "post" route. The route can be used to show ***supplementary*** angles.

6. Tell what it means for angles to be supplementary and sketch an example below.
Two angles are supplementary if the sum of their measures is 180°.

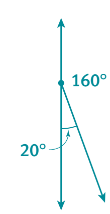


7. This "post" route is seen below as it passes over the goal line. Give the measure of angle *d*. **70°**

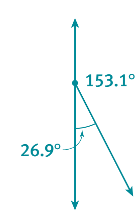


8. Coach Toose's team runs a variety of "post" routes. Identify the angle supplementary to the one listed. Draw a diagram to illustrate the angle and its supplement.

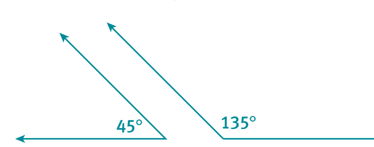
a. 20°
160°



b. 153.1°
26.9°



9. One of Coach Toose's players claims that two angles do not need to be adjacent to be supplementary. Draw a pair of nonadjacent supplementary angles and explain why they are supplementary.
Sample answer: An angle with measure 45° and an angle with measure 135° are supplementary because 45 + 135 = 180.



Lesson 16-1
Complementary and Supplementary Angles

My Notes

10. The measures of two supplementary angles are $(x + 1)°$ and $(2x - 1)°$.

a. Write an equation that shows the relationship between the two angle measures and determine the value of x.
$x + 1 + 2x - 1 = 180$
$x = 60$

b. Determine the measure of the two angles. **61°; 119°**

Check Your Understanding

11. Determine the complement and/or supplement of each angle. If it is not possible, explain.
a. 57.2° **b.** 93°

12. Determine whether angles with measures 47° and 53° are complementary. Explain why or why not.

13. Determine whether angles with measures 37° and 143° are supplementary. Explain why or why not.

LESSON 16-1 PRACTICE

14. Determine the measure of two ***congruent***, complementary angles.

15. Draw a pair of adjacent, complementary angles.

16. Angle C and angle D are complementary. The measure of angle C is $(2x)°$ and the measure of angle D is $(3x)°$. Determine the measure of the two angles. Show the work that leads to your answer.

17. Angle E and angle F are supplementary. The measure of angle E is $(x + 10)°$ and the measure of angle F is $(x + 40)°$. Determine the measure of the two angles. Show the work that leads to your answer.

18. **Construct viable arguments.** Determine whether the following statement is true or false. Justify your reasoning. "Two right angles are always supplementary."

MATH TERMS

Angles that have the same measure are called **congruent**.

ACTIVITY 16 Continued

Check Your Understanding

These items serve as a formative assessment of students' understanding of complementary and supplementary angles. Have students debrief their answers and remind them to share their reasoning. Hearing other students' thought processes will help struggling students develop their understanding of the key concepts.

Answers

11. a. complement: 32.8°; supplement: 122.8°
b. complement: not possible because the measure of the given angle is greater than 90°; supplement: 87°

12. no; $43° + 57° = 100° \neq 90°$

13. yes; $37° + 143° = 180°$

ASSESS

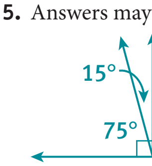
Use the Lesson Practice to assess students' understanding of complementary and supplementary angles.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 16-1 PRACTICE

14. 45°

15. Answers may vary.



16. $2x + 3x = 90$; $5x = 90$; $x = 18$; $m\angle C = 2(18) = 36°$; $m\angle D = 3(18) = 54°$

17. $x + 10 + x + 40 = 180$; $2x + 50 = 180$; $2x = 130$; $x = 65$; $m\angle E = 65 + 10 = 75°$; $m\angle F = 65 + 40 = 105°$

18. True. Explanations may vary. All right angles measure 90°, and $90° + 90° = 180°$.

ADAPT

Check students' work to be sure they are able to work with complementary and supplementary angles in a variety of mathematical contexts. Students who need additional practice will gain further experience with these concepts in Lesson 16-2. You may also want to assign additional problems from the Activity Practice.

Lesson 16-2

PLAN

Materials

- masking tape
- protractor

Pacing: 2 class periods

Chunking the Lesson

#1–6 #7–8
Check Your Understanding
#12–15 #16–19
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity

Ask students to identify a pair of complementary angles and a pair of supplementary angles in the classroom. Encourage students to consider windows, walls, and patterns of floor tiles. Then have students share their findings. Explain that they will learn about some new angle pairs in this lesson.

1–6 Use Manipulatives, Visualization, Predict and Confirm, Think-Pair-Share, Look for a Pattern, Discussion Groups Asking students to classify the angles as acute or obtuse in their tape diagrams will help them to identify congruent angles. In Item 3, monitor students to be sure they are making their predictions without measuring. It is important for students to recognize that there are only two different angle measures in the diagram. Ask students to think individually about the relationships in the diagram before discussing with a partner and/or cooperative group. During the share-and-respond time, be sure each student recognizes that each pair of angles in the diagram is either congruent or supplementary.

Teacher to Teacher

Students will use tape on their desks (or on a large sheet of paper) as a manipulative for creating parallel lines cut by a transversal. In order for them to recognize key angle relationships, be sure students do not create a transversal perpendicular to the parallel lines. The angles may be labeled with sticky notes or small pieces of tape. In order to conserve tape, you might have a pair or trio of students work together to create one diagram.

Lesson 16-2
Angles Formed by Parallel Lines

My Notes

MATH TERMS

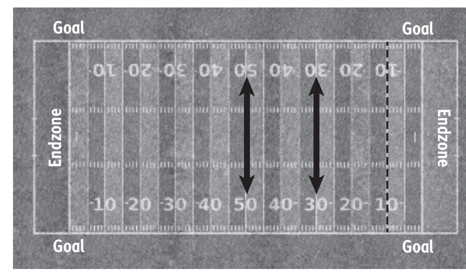
A **transversal** is a line that intersects two or more other lines to form eight or more angles.

Learning Targets:

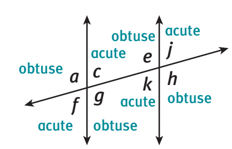
- Determine the measure of angles formed by parallel lines and transversals.
- Identify angle pairs formed by parallel lines and transversals.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Think-Pair-Share, Interactive Word Wall, Graphic Organizer

The coach uses a diagram like the one below to show plays to his team. Your teacher will give you tape to recreate these same play lines on your desk or on a piece of paper.



Now using the tape, add a "slant" route to your diagram and label the angles as seen below.



Coach Toose calls this route the "***transversal***."

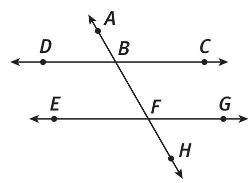
1. On the above diagram, mark each of the eight angles formed by the parallel lines and the transversal as acute or obtuse.
2. Measure angle *j* on your diagram.
   In the above diagram, angle *j* measures 75°.
3. Without measuring, predict which other angles are the same size as angle *j* and list them below.
   k*, *c*, and *f
4. Now measure these angles. Were your predictions correct?
   Yes; angles *k*, *c*, and *f* have the same measure as angle *j*.
5. What is true about the measures of the remaining angles?
   The remaining angles are the same size or congruent.

Lesson 16-2
Angles Formed by Parallel Lines

My Notes

6. Using the diagram that you made on your desk and your observations in the previous items, what can you say about the measures of the angles formed by two parallel lines cut by a transversal?
Sample answer: Parallel lines cut by a transversal create two sets of angles that have the same measure. There are also supplementary angles in the diagram.

In the diagram, $\overline{CD} \parallel \overline{EG}$.



7. Determine whether each pair of angles is congruent or supplementary.
- **a.** $\angle ABD$ and $\angle CBH$ **congruent**
- **b.** $\angle ABD$ and $\angle EFH$ **supplementary**
- **c.** $\angle DBH$ and $\angle CBF$ **supplementary**
- **d.** $\angle ABC$ and $\angle BFG$ **congruent**
- **e.** $\angle CBF$ and $\angle EFB$ **congruent**

8. Critique the reasoning of others. In the diagram, $m\angle CBF = (x + 10)°$ and $m\angle EFB = (3x - 54)°$. Students were asked to determine the value of x. One student's solution is shown below. Determine whether or not the solution is correct. If it is correct, explain the reasoning used by the student. If it is incorrect, identify the student's error and explain to the student how to correctly solve the problem.

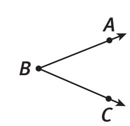
$$\begin{aligned} x + 10 + 3x - 54 &= 180 \\ 4x - 44 &= 180 \\ 4x &= 224 \\ x &= 56 \end{aligned}$$

The student's solution is incorrect.
Sample explanation: $\angle CBF$ and $\angle EFB$ are not supplementary. They have the same measure, so you should set the expressions equal to each other to solve. $x + 10 = 3x - 54$; $10 = 2x - 54$; $64 = 2x$; $32 = x$

READING MATH

The symbol $\parallel$ is used to indicate parallel lines. $\overleftrightarrow{CD} \parallel \overleftrightarrow{EG}$ is read "line *CD* is parallel to line *EG*."

READING MATH



To read this angle, say "angle *ABC*," "angle *CBA*," or "angle *B*."

ACTIVITY 16 Continued

7–8 Think-Pair-Share, Group Presentation In Item 8, students are asked to examine a written solution provided by a student and critique the reasoning in the solution. Students should be precise in explaining the required correction for the solution. Monitor students' discussions and presentations to evaluate whether they are using mathematical terms and academic vocabulary correctly and whether they have internalized meanings well enough to demonstrate fluency in their discussions. Groups can share their response to this item on whiteboards. It is important for students to recognize how to correctly set up the equation. They should be able to distinguish when the expressions for the angle measures should be set equal and when they should be added to equal 180.

Developing Math Language

Students are introduced to the term *transversal* in this lesson. Remind students to refer to the English-Spanish glossary to aid them in comprehending new vocabulary words.

Check Your Understanding

In Item 9, some students may need to measure each of the angles with a protractor, some may measure a few of the angles and be able to determine the rest of the measures, and other students may be able to take the given measure and complete the table without using a protractor at all. Allow students to complete the table using whatever method they are comfortable with. When they are done, they can compare methods with the other students in the class. This will help move all students to the level of understanding that allows them to complete the problem without a protractor.

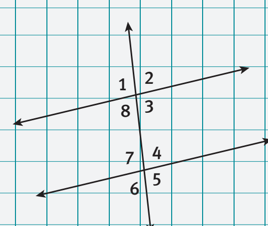
Answers

9.

Angle	Measure
∠*RHC*	125°
∠*JHK*	125°
∠*RHJ*	55°
∠*CHK*	55°
∠*HKS*	55°
∠*SKU*	125°
∠*OKU*	55°
∠*HKO*	125°

10. 60
11. 23

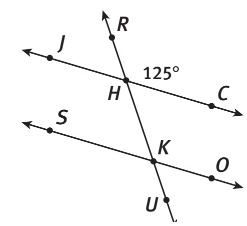
My Notes



Check Your Understanding

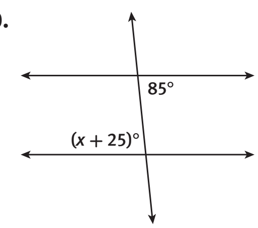
9. In the diagram, $\overline{JC} \parallel \overline{SO}$. Copy and complete the table to find the missing angle measures.

Angle	Measure
∠*RHC*	125°
∠*JHK*	
∠*RHJ*	
∠*CHK*	
∠*HKS*	
∠*SKU*	
∠*OKU*	
∠*HKO*	

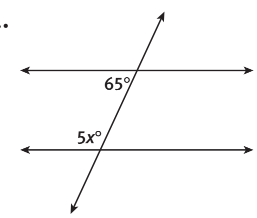


Each diagram shows parallel lines cut by a transversal. Solve for the value of *x*.

10.



11.



12. Refer to the diagram in the My Notes section.

a. What does the term *exterior* mean in everyday language? Give at least two examples.
Sample answer: *Exterior* means outside. Examples: exterior paint, exterior furniture, and exterior color on a car

b. Which angles in the figure do you think are exterior angles? Explain.
∠1, ∠2, ∠5, ∠6; They are the angles on the outside.

c. What does the term *interior* mean in everyday language? Give at least two examples.
Sample answer: *Interior* means inside. Examples: interior paint, a car's interior, and interior design

d. Which angles in the figure do you think are interior angles? Explain.
∠3, ∠4, ∠7, ∠8; They are the angles on the inside.

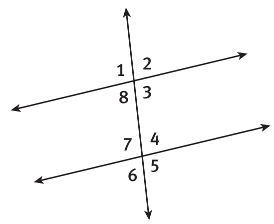
Lesson 16-2
Angles Formed by Parallel Lines

My Notes

13. Alternate angles are on opposite sides of the transversal and have a different vertex. There are two pairs of angles in the diagram that are referred to as ***alternate exterior angles*** and two pairs of angles that are referred to as ***alternate interior angles***.

a. Explain what it means for angles to be *alternate exterior angles.*

Alternate exterior angles are on opposite sides of the transversal line, and outside the parallel lines.



b. Name the two pairs of alternate exterior angles in the diagram.

$\angle 1$ & $\angle 5$ and $\angle 2$ & $\angle 6$

ACADEMIC VOCABULARY

As a verb, the word ***alternate*** means to shift back and forth between one state and another.

c. Explain what it means for angles to be *alternate interior angles.*

Alternate interior angles are on opposite sides of the transversal line, and inside the parallel lines.

d. Name the two pairs of alternate interior angles in the diagram.

$\angle 3$ & $\angle 7$ and $\angle 4$ & $\angle 8$

14. The above diagram shows a pair of parallel lines cut by a transversal.

a. If the measure of $\angle 2$ is 70°, determine the measures of the other angles.

$m\angle 1 = m\angle 3 = m\angle 5 = m\angle 7 = 110°$

$m\angle 2 = m\angle 4 = m\angle 6 = m\angle 8 = 70°$

b. What relationship do you notice about the measures of the alternate exterior angles?

They have the same measure.

c. What relationship do you notice about the measures of the alternate interior angles?

They have the same measure.

ACTIVITY 16 Continued

12–15 Visualization, Interactive Word Wall, Graphic Organizer As a class, discuss the various examples students have chosen for *exterior* and *interior*. Students should use their definitions for *exterior* and *interior* to identify angles in the diagram. Before beginning Item 13, some students may benefit from discussing the term *alternate*. Students may be able to think of real world examples for this term as well.

Differentiating Instruction

Support Visual learners may benefit from using colored pencils or highlighters to color in the angle pairs so that the relationships are obvious.

Extend Ask students to sketch three parallel lines that are cut by a transversal and have them identify pairs of alternate interior angles, alternate exterior angles, and corresponding angles.

Developing Math Language

This lesson contains several new terms that students will encounter again and again in future math courses. Monitor students' understanding of what they read by asking key questions about information provided or the meaning of unknown or difficult words.

16–19 Visualization, Think-Pair-Share, Sharing and Responding This is another opportunity for students to determine the measures of unknown angles. Students need to determine the unknown angle measures without using a protractor. Students should notice that each pair of alternate interior angles are congruent, each pair of alternate exterior angles are congruent, each pair of corresponding angles are congruent, and each pair of vertical angles are congruent.

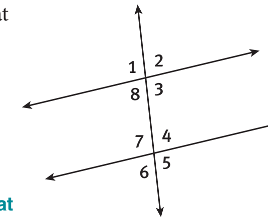
Teacher to Teacher

There are many new geometric terms in this activity. Some students may have difficulty remembering and applying these terms. The following ideas may be helpful.

- Have students design flashcards for these terms and hook the cards together with a shower curtain ring to keep them together for studying.
- Have students create a bingo game with terms and definitions. Then play the game with the class.
- Play "term charades," with students writing down their guesses on whiteboards for the term that is acted out.
- Have students make a concentration-type game with definitions, examples, and terms as the matching items.

My Notes

15. Another classification for angle pairs that exist when two lines are cut by a transversal is ***corresponding angles***. In the diagram, $\angle 2$ and $\angle 4$ are corresponding.



a. What do you think is meant by the term *corresponding*?

Corresponding angles are angles that are at the same location at each intersection in the figure.

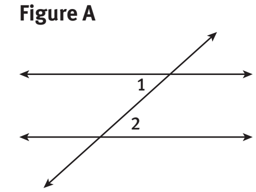
b. Name the three other pairs of corresponding angles in the diagram and tell what you notice about the measures of these angles.

$\angle 1$ & $\angle 7$, $\angle 8$ & $\angle 6$, $\angle 3$ & $\angle 5$. The angles have the same measure.

16. In Figure A, two parallel lines are cut by a transversal. The measure of $\angle 1 = 42°$. Find $m\angle 2$ and describe the relationship that helped you determine the measure.

$m\angle 2 = 42°$. Sample explanation: Alternate interior angles are congruent when two parallel lines are cut by a transversal.

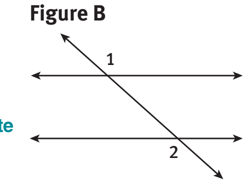
Figure A



17. In Figure B, two parallel lines are cut by a transversal. The measure of $\angle 1 = 138°$. Find $m\angle 2$ and describe the relationship that helped you determine the measure.

$m\angle 2 = 138°$. Sample explanation: Alternate exterior angles are congruent when two parallel lines are cut by a transversal.

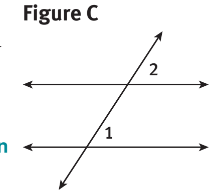
Figure B



18. In Figure C, two parallel lines are cut by a transversal. The measure of $\angle 1 = 57°$. Find $m\angle 2$ and describe the relationship that helped you determine the measure.

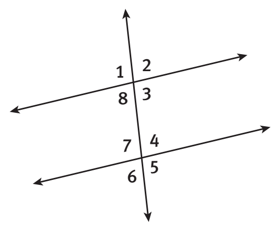
$m\angle 2 = 57°$. Sample explanation: Corresponding angles are congruent when two parallel lines are cut by a transversal.

Figure C



My Notes

19. Two pairs of ***vertical angles*** are formed when two lines intersect. They share a vertex but have no common rays. List the pairs of vertical angles in the diagram and tell what you notice about the measures of these angles.

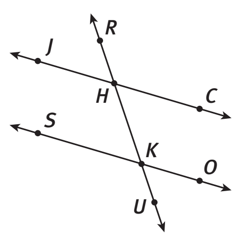


$\angle 1$ & $\angle 3$, $\angle 2$ & $\angle 8$, $\angle 4$ & $\angle 6$, $\angle 7$ & $\angle 5$.
They have the same measure.

Check Your Understanding

20. Identify each pair of angles as alternate interior, alternate exterior, corresponding, or vertical.

a. $\angle RHJ$ and $\angle RKS$
b. $\angle CHK$ and $\angle SKH$
c. $\angle OKU$ and $\angle HKS$
d. $\angle CHK$ and $\angle UKO$
e. $\angle RHJ$ and $\angle OKU$



21. Angles ABC and ADF are alternate interior angles. The measure of $\angle ABC = (8x + 4)°$ and the measure of $\angle ADF = (10x - 20)°$. Determine the measure of each of the angles.

Check Your Understanding

Debrief students' answers to these items to assess their understanding of alternate interior angles, alternate exterior angles, corresponding angles, and vertical angles. Encourage students to use complete sentences in their responses.

Answers

20. a. corresponding
b. alternate interior
c. vertical
d. corresponding
e. alternate exterior

21. $m\angle ABC = m\angle ADF = 100°$

ASSESS

Use the Lesson Practice to assess the students' understanding of the angles formed when parallel lines are cut by a transversal.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 16-2 PRACTICE

22. $\angle 1$ & $\angle 7$, $\angle 2$ & $\angle 8$, $\angle 3$ & $\angle 5$, $\angle 4$ & $\angle 6$

23. $\angle 1$, $\angle 3$, $\angle 5$, $\angle 7$

24. $m\angle 3 = 123°$; $m\angle 4 = 57°$

25. $m\angle 6 = m\angle 8 = 61°$

26. 63°

27. Answers will vary. Yes; since the corresponding angles are congruent, the four angles formed by the transversal and the other parallel line must also be right angles.

28. Answers will vary. Let $m\angle D = x$ and $m\angle A = 5x$. Since $\angle A$ and $\angle D$ are supplementary, $x + 5x = 180$, so $6x = 180$ and $x = 30$. Therefore, $m\angle D = 30°$ and $m\angle A = 150°$.

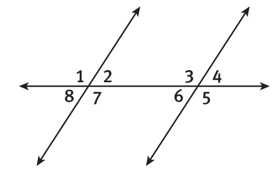
ADAPT

The lesson practice serves as a formative assessment of students' understanding of the angle pairs that are formed when parallel lines are cut by a transversal. Assign additional problems from the Activity Practice as needed if students require further work with this topic.

My Notes

LESSON 16-2 PRACTICE

The figure shows a pair of parallel lines that are intersected by a transversal. Use the figure for Items 22–26.



22. Name all pairs of vertical angles in the figure.

23. Name all of the angles in the figure that are supplementary to $\angle 8$.

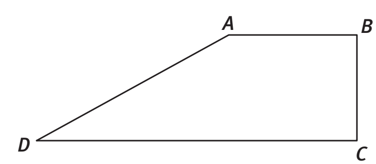
24. If $m\angle 2 = 57°$, find $m\angle 3$ and $m\angle 4$.

25. If $m\angle 6 = (5x + 1)°$ and $m\angle 8 = (7x - 23)°$, find $m\angle 6$ and $m\angle 8$.

26. Suppose $\angle 9$, which is not shown in the figure, is complementary to $\angle 4$. Given that $m\angle 1 = 153°$, what is $m\angle 9$?

27. Two parallel lines are intersected by a transversal. The transversal forms four right angles with one of the parallel lines. Can you conclude that the transversal forms four right angles with the other parallel line? Justify your answer.

28. **Model with mathematics.** DeMarco is designing a skateboard ramp as shown in the figure. He wants the sides $\overline{AB}$ and $\overline{CD}$ to be parallel to each other. He also wants the measure of $\angle A$ to be five times the measure of $\angle D$. Explain how he can find the correct measures of these two angles.



ACTIVITY 16 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 16-1

1. Are angles with measures of 11° and 89° complementary? Why or why not?

2. Can two obtuse angles be supplementary? Explain why or why not.

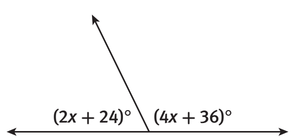
3. What is the measure of an angle that is supplementary to an angle that measures 101°?

4. What is the measure of an angle that is complementary to an angle that measures 75°?

5. The measures of two complementary angles are $(3y - 1)°$ and $(4y + 7)°$.
a. Determine the value of y.
b. Calculate the measure of each of the angles.

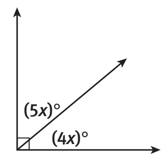
6. The measures of two supplementary angles are $\left(\frac{1}{2}x\right)°$ and $(x + 30)°$.
a. Determine the value of x.
b. Calculate the measure of each of the angles.

7. In the figure below, determine the value of x.



8. Suppose $\angle A$ is complementary to $\angle B$ and $\angle B$ is supplementary to $\angle C$. If $m\angle A$ is 21°, find $m\angle C$.

9. A student determined the value of x as shown. Explain the student's error.



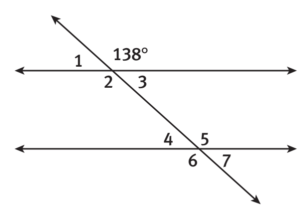
$$4x + 5x = 180$$
$$9x = 180$$
$$x = 20$$

10. $\angle 1$ and $\angle 2$ are supplementary. Which of the following statements cannot be true?
A. $\angle 1$ is obtuse and $\angle 2$ is acute.
B. $\angle 1$ and $\angle 2$ are adjacent angles.
C. $\angle 1$ and $\angle 2$ are congruent angles.
D. $\angle 1$ and $\angle 2$ are complementary.

11. $\angle A$ and $\angle B$ are complementary angles. The measure of $\angle A$ is 4 times the measure of $\angle B$. Which of these is the measure of $\angle B$?
A. 18° **B.** 22.5°
C. 36° **D.** 72°

Lesson 16-2

12. The diagram below shows parallel lines cut by a transversal. Determine the measures of $\angle 1$ through $\angle 7$.

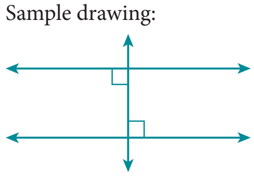


13. Name a pair of alternate interior angles in the above figure.

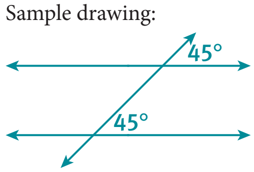
ACTIVITY PRACTICE

1. Angles with measures of 11° and 89° are not complementary because the sum of these angle measures is not 90°.
2. Two obtuse angles cannot be supplementary because the sum of their measures would be greater than 180°.
3. 79°
4. 15°
5. **a.** $y = 12$
b. 35° and 55°
6. **a.** $x = 100$
b. 50° and 130°
7. $x = 20$
8. 111°
9. Answers may vary. The sum of the angle measures should be 90° rather than 180°.
10. D
11. A
12. $m\angle 1 = m\angle 3 = m\angle 4 = m\angle 7 = 42°$; $m\angle 2 = m\angle 5 = m\angle 6 = 138°$
13. Answers may vary. $\angle 3$ and $\angle 4$

14. a. 82° because the angles are alternate interior angles.
b. 98° because ∠2 and ∠7 are supplementary angles.
c. 82° because ∠2 and ∠4 are corresponding angles.
d. 82° because ∠2 and ∠8 are vertical angles.
15. $y = 21$
16. $w = 49$
17. Sample drawing:



18. Sample drawing:

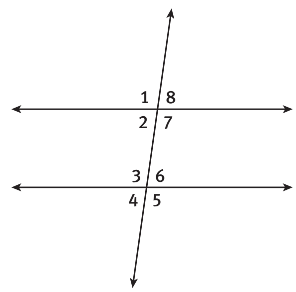


19. a. 69°
b. 45
20. sometimes
21. always
22. never
23. always
24. Yes; since $\overline{ED}$ is parallel to $\overline{AF}$, $m\angle BAC = 50°$ since $\angle EBA$ and $\angle BAC$ are alternate interior angles. This means $m\angle DCF = 50°$ also, since $\overline{AB} \parallel \overline{CD}$ and $\angle BAC$ and $\angle DCF$ are corresponding angles.

ADDITIONAL PRACTICE

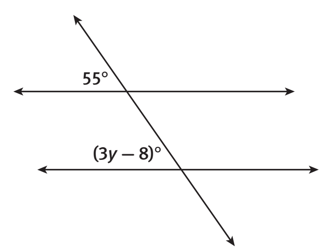
If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.

14. Two parallel lines are cut by a transversal as shown below. Find each of the following measures if $m\angle 2 = 82°$. Explain your answers.

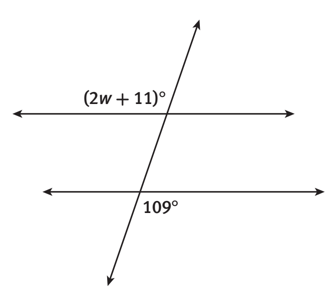


a. $m\angle 6$ **b.** $m\angle 7$
c. $m\angle 4$ **d.** $m\angle 8$

15. The figure shows parallel lines cut by a transversal. Find the value of y.



16. The figure shows parallel lines cut by a transversal. Find the value of w.

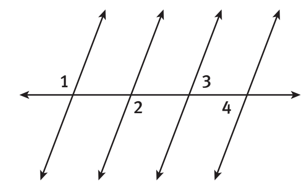


Draw and label a figure to match each statement.

17. Parallel lines are cut by a transversal and a pair of alternate interior angles are right angles.

18. Parallel lines are cut by a transversal and a pair of corresponding angles are complementary.

19. The figure shows several parking spaces at a mall. The parking spaces were created by drawing four parallel lines and a transversal.



a. If $m\angle 1 = 111°$, find $m\angle 4$.
b. If $m\angle 2 = (3x - 15)°$ and $m\angle 3 = (2x - 30)°$, find the value of x.

Determine whether each statement is always, sometimes, or never true.

20. When two parallel lines are intersected by a transversal, all of the corresponding angles are right angles.

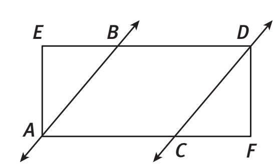
21. When two parallel lines are intersected by a transversal, every pair of angles are either congruent or supplementary.

22. When two parallel lines are intersected by a transversal, there is one pair of alternate exterior angles that are not congruent.

23. If $\angle Q$ and $\angle R$ are vertical angles, then $\angle Q$ is congruent to $\angle R$.

MATHEMATICAL PRACTICES
Make Sense of Problems

24. The figure shows rectangle *EDFA*. The opposite sides of the rectangle are parallel. Also, $\overline{AB} \parallel \overline{CD}$. If $m\angle EBA = 50°$, is it possible to determine $m\angle DCF$? If so, explain how. If not, explain why not.



Angles of Triangles and Quadrilaterals

ACTIVITY 17

The Parallel Chute
Lesson 17-1 Angles in a Triangle

My Notes

Learning Targets:

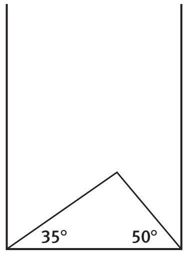
- Describe the relationship among the angles of a triangle.
- Write and solve equations involving angles of a triangle.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Graphic Organizer, Create Representations, Think-Pair-Share

Chip designs games for his computer. One of his current projects is called Parallel Chute. In the game, triangles appear at the top of a long chute that has parallel sides and slowly descend to rest at the bottom. The object of the game is to completely fill the region between the parallel sides of the chute.

As the triangle descends in the chute, the player is allowed to change the position of the triangle with the following commands: flip, slide, and turn.

The figure shows a triangle that has come to rest at the bottom of the chute.



1. Work with your group. Use the triangular pieces given to you by your teacher to fill in the rectangular chute you have. Is there more than one way to fill the chute? Explain.

   Yes; explanations may vary.

Common Core State Standards for Activity 17

8.G.A.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

ACTIVITY 17
Investigative

Activity Standards Focus

In this activity, students investigate and apply the fact that the sum of the measures of the angles in a triangle is 180°. Students also describe the relationship between an exterior angle of a triangle and its remote interior angles, and the relationship among the angles of a quadrilateral.

Lesson 17-1

PLAN

Materials

- blackline masters (Empty Chute and Triangular Pieces)
- scissors

Pacing: 1–2 class periods

Chunking the Lesson

#1 #2–5 #6–8
#9–10 #11
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity

Give students a few minutes to write down the facts that they know about triangles, encouraging them to use appropriate mathematical terminology. Have volunteers debrief their lists with the class, and explain that students will explore an important property of triangles in this lesson.

Teacher to Teacher

Each pair of students will need a copy of the empty chute and the triangular puzzle pieces. One set per pair of students should suffice. The blackline masters for the chute and puzzle pieces can be found in the back of the book. The triangular pieces can be cut out before class to save time, or students can be asked to cut them out in class.

1 Use Manipulatives, Create Representations, Shared Reading, Summarizing Students should use the triangular pieces to fill in the chute. This hands-on part of the activity should help students understand the importance of placing angles whose measures add up to 180° adjacent to each other along the sides of the chute.

2–5 Look for a Pattern, Think-Pair-Share, Sharing and Responding, Identify a Subtask Students are expected to recognize complementary angles ($\angle CAD$ and $\angle CAB$) and alternate interior angles ($\angle CAD$ and $\angle ACB$). Because the sides of the chute are parallel, the alternate interior angles have the same measure. It is essential that you discuss this item with the class before they continue with Items 3–5. Students should be encouraged to edit their written responses once this item has been discussed.

In Item 3, the measure of $\angle ECD$ is 79° because the three angles with vertex C must add up to 180°. Students should use congruent alternate interior angles to find the measure of $\angle CDA$. $\angle FDC$ is the supplement to $\angle CDA$.

Students should realize that the three angles in $\triangle ACD$ have the same measure as the three angles whose vertex is at C.

Teacher to Teacher

Students may benefit from labeling angle measures on the diagrams as they determine the measures. Students will continue to work with these measures in subsequent problems.

ELL Support

Visual learners may find it helpful to use colored pencils to help identify the pairs of alternate interior angles in the diagram.

ACTIVITY 17
continued

My Notes

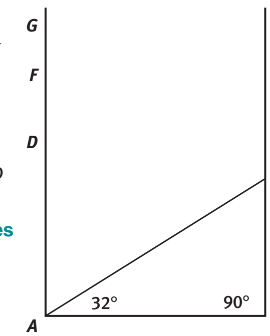
Lesson 17-1
Angles in a Triangle

During the game, before each triangle appears, the player must select the measure (in degrees) of one angle in the triangle.

2. In one game, Chip's first triangle with a 90° angle came to rest and displayed the measure of $\angle CAB$ to be 32°.

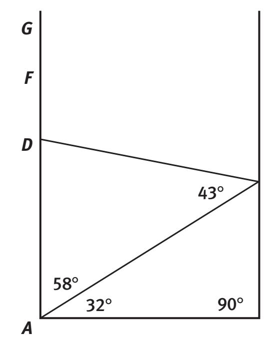
a. Determine the measure of $\angle CAD$.
58°

b. Explain why the measure of $\angle CAD$ must equal the measure of $\angle ACB$.
Sample answer: If two parallel lines are cut by a transversal, then alternate interior angles are congruent.



3. When $\triangle ACD$ came down the chute, Chip selected the 58° angle and the computer selected the 43° angle. Determine the measure of each of the following angles.

a. $\angle ECD$ 79°
b. $\angle CDA$ 79°
c. $\angle FDC$ 101°



4. List the measures of the three angles in $\triangle ACD$. Then list the measures of the three nonoverlapping angles whose vertex is at C. How do the two lists compare?

Measures of angles in triangle *ACD*: 79°, 58°, and 43°

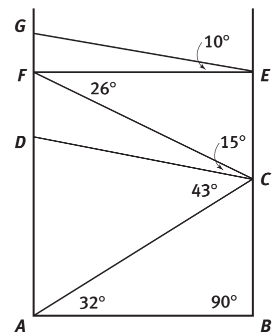
Measures of the nonoverlapping angles whose vertex is at *C*: 79°, 58°, and 43°

The lists are the same.

Lesson 17-1
Angles in a Triangle

My Notes

5. Find the measure of each of the following angles.
a. $\angle FCE$ **64°**
b. $\angle CFD$ **64°**
c. $\angle EFG$ **90°**
d. $\angle CEF$ **90°**
e. $\angle FGE$ **80°**



6. Every triangle has three sides and three angles. Use your responses to Items 2, 3, and 5 to complete the following table. For each triangle, list the angle measures and find the sum of the measures of the three angles.

Triangle Name	Angle Measures	Sum of Angle Measures
△ *ABC*	90°, 32°, 58°	**180°**
△ *ACD*	**58°, 43°, 79°**	**180°**
△ *DCF*	**101°, 15°, 64°**	**180°**
△ *ACF*	**64°, 58°, 58°**	**180°**
△ *CEF*	**90°, 26°, 64°**	**180°**
△ *GEF*	**90°, 80°, 10°**	**180°**

7. **Express regularity in repeated reasoning.** Write a conjecture about the sum of the measures of the angles of a triangle.
The sum of the measures of the angles of a triangle is 180°.

ACTIVITY 17 Continued

6–7 Look for a Pattern, Graphic Organizer, Think-Pair-Share, Sharing and Responding, Activating Prior Knowledge In Item 6, students list the measures of the three angles in six different triangles and calculate the sum of the three angles for each triangle. Using the information gathered in the table, students write a conjecture about the sum of the measures of the angles. Some students may remember this fact from previous math courses.

TEACHER to TEACHER

Monitor students' writing to ensure that they are using language correctly, including adequate details, and describing mathematical reasoning using precise terms. For example, many students may write "the sum of the angles of a triangle is 180°." Remind students that they should write "the sum *of the measures* of the angles of a triangle is 180°."

8 Think-Pair-Share, Sharing and Responding, Activating Prior Knowledge In Item 8, students are asked to give an informal argument using transversals and parallel lines explaining why the sum of the measures of the angles in a triangle is 180°. Sharing and Responding can be used to quickly assess student understanding before moving on to the next item.

9–10 Think-Pair-Share, Sharing and Responding, Identify a Subtask In Item 9, students are given an opportunity to practice calculating an unknown angle measure in a triangle (by adding the measures of the two known angles and subtracting from 180°). In Item 10, students must apply what they have learned about the sum of the measures of the angles in a triangle to critique someone's reasoning and find an error in a statement. It is helpful to have groups share their responses to this item for comparison.

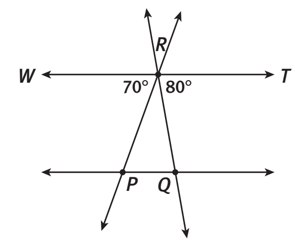
ELL Support

Support To support ELL students in reading the problem scenario in Item 10, carefully group students to ensure that all students participate and have an opportunity for meaningful reading and discussion. Suggest that group members each read a sentence and explain what that sentence means to them. Group members can then confirm one another's understanding of the key information provided for the problem.

Extend Give students an opportunity to write their own word problem based on the fact that the sum of the measures of the angles of a triangle is 180°.

My Notes

8. In the diagram, $\overleftrightarrow{WT} \parallel \overleftrightarrow{PQ}$.



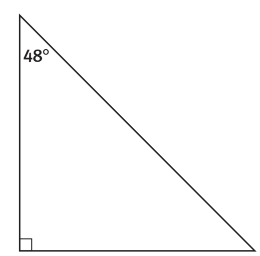
a. Use what you know about parallel lines and transversals to determine the measures of $\angle RPQ$ and $\angle RQP$.

$m\angle RPQ = 70°$, $m\angle RQP = 80°$

b. Explain how this diagram supports your conjecture in Item 7.

The measure of $\angle PRQ = 30°$. Since $m\angle RPQ + m\angle RQP + m\angle PRQ = 70° + 80° + 30° = 180°$, the measures of the angles of the triangle add up to 180°.

9. Determine the measure of the unknown angle in the triangle below. 42°



My Notes

10. Chip has discovered an error in the programming of the game. Before a triangle appeared, a player selected an angle with measure 100° and the computer selected 82° for a different angle measure. Explain how Chip knew there was an error.

Sample answer: The two angle measures given add up to 182°, but the three angle measures of a triangle must add up to 180°. The sum of the first two angle measures is already more than 180°.

11. The measures of the three angles in a triangle are $x°$, $(2x + 4)°$ and $(2x - 9)°$.

a. Write an equation based on the relationship between the three angle measures and then solve for x.

$$x + 2x + 4 + 2x - 9 = 180$$
$$x = 37$$

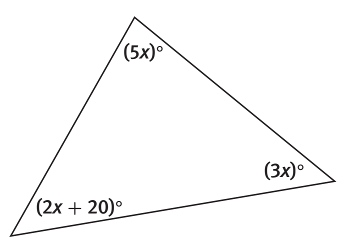
b. Determine the measures of the three angles of the triangle.

37°, 65°, 78°

Check Your Understanding

12. If one of the acute angles of a right triangle has a measure of 22°, calculate the measure of the other acute angle.

13. Suppose the measures of the angles in a triangle are given in the figure. Write an equation, solve for x, and determine the measure of each angle.



ACTIVITY 17 Continued

11 Think-Pair-Share, Create Representations, Identify a Subtask

This item takes students to a more abstract level. Once students write and solve an equation to determine the value of x, they must substitute the value of x into the expression for each angle's measure. Students can check their answers by calculating the sum of the angle measures and making sure the sum is 180°.

Check Your Understanding

Students can complete these items individually or in groups. Debrief students' results to ensure that they know how to solve problems about the angle measures in a triangle.

Answers

12. 68°

13. $2x + 20 + 3x + 5x = 180$; $x = 16$; the angle measures are 48°, 52°, and 80°.

ASSESS

Use the Lesson Practice to assess students' understanding of finding the measures of angles in a triangle.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 17-1 PRACTICE

14. 79°
15. 35°
16. $4x + 3x - 3 + 5x + 3 = 180$; $x = 15$; the angle measures are 42°, 60°, and 78°.
17. 45°, 45°, and 90°
18. It is not possible because the sum of the measures of the two right angles is 180°, so the sum of the measures of all three angles of the triangle would be greater than 180°.
19. a. Yes; 38°, 52°, and 90°
b. Since the acute angle formed by the brace and the shelf measures 52°, the obtuse angle measures $180° - 52° = 128°$.

ADAPT

Check students' work to be sure they are able to solve problems about angles in a triangle. Students who need additional practice with this skill will have further opportunities to work with angles in triangles in Lesson 17-2.

My Notes

LESSON 17-1 PRACTICE

In Items 14 and 15, the measures of two angles of a triangle are given. Find the measure of the third angle of the triangle.

14. 23°, 78°

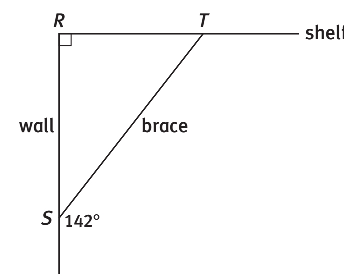
15. 105°, 40°

16. The measures of the three angles in a triangle are $(4x)°$, $(3x - 3)°$, and $(5x + 3)°$. Write an equation, solve for x, and determine the measure of each angle.

17. In $\triangle ABC$, $\angle A$ and $\angle B$ have the same measure. The measure of $\angle C$ is twice the measure of $\angle A$. Find the measures of the angles in the triangle.

18. Eliana claimed that she drew a triangle with two right angles. Draw a sketch of such a triangle or explain why it is not possible.

19. **Model with mathematics.** Brian is building a brace for a shelf. The figure shows the plans for the brace.



a. Based on the information given in the figure, is it possible to determine the three angles of $\triangle RST$? If so, find the measures. If not, explain why not.

b. Brian wants to know the measure of the obtuse angle formed by the brace and the shelf. Explain how he can determine this.

Lesson 17-2
Exterior Angles and Angles in Quadrilaterals

My Notes

Learning Targets:

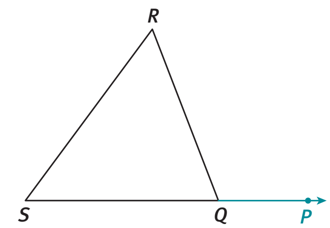
- Describe and apply the relationship between an exterior angle of a triangle and its remote interior angles.
- Describe and apply the relationship among the angles of a quadrilateral.

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Look for a Pattern, Visualization, Create Representations, Think-Pair-Share

An ***exterior angle*** of a triangle is formed by extending a side of the triangle. The vertex of the exterior angle is a vertex of the triangle. The sides of the exterior angle are determined by a side of the triangle and the extension of the adjacent side of the triangle at the vertex.

1. Use $\triangle SRQ$ below.

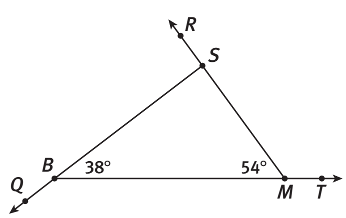
a. Extend side $\overline{SQ}$ of the triangle by drawing $\overrightarrow{SP}$ through point Q to create exterior angle RQP.



b. Describe the relationship between the measures of $\angle RQP$ and $\angle RQS$.
The angles are supplementary.

2. An exterior angle has been drawn at each of the three vertices of $\triangle SBM$.

a. Determine the measure of each of the three exterior angles.
$m\angle SMT = 126°$, $m\angle RSB = 92°$, $m\angle QBM = 142°$



Lesson 17-2

PLAN

Pacing: 1–2 class periods

Chunking the Lesson

#1–2
Check Your Understanding
#6–8
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity

Ask students to determine whether a right triangle can include angles that measure 43° and 27°. Prompt students to write at least one complete sentence explaining their reasoning, then debrief students' answers. Tell students they will continue to work with angles and triangles in this lesson.

1–2 Shared Reading, Visualization, Marking the Text, Vocabulary Organizer, Create Representations, Sharing and Responding You or a student can read aloud the introduction to Item 1 while other students read along and mark important information. Students should identify the relationship between an exterior angle and the interior angle with the same vertex as adjacent, supplementary angles. It is important for students to recognize this relationship before moving on to Item 2.

Developing Math Language

This lesson introduces the terms *exterior angle* and *remote interior angle.* As students respond to questions or discuss possible solutions to problems, monitor their use of these new terms and their descriptions of applying math concepts to ensure their understanding and ability to use language correctly and precisely.

Check Your Understanding

These items serve as a formative assessment to check whether students can apply the Exterior Angle Theorem in a variety of settings. Be sure to spend a few minutes debriefing students' work before moving on to considering angles in quadrilaterals.

Answers

3. 115°

4. $m\angle NYX = 112°$, $m\angle RXA = 111°$, $m\angle PAY = 137°$

5. 140°

My Notes

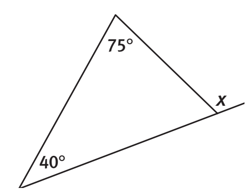
b. For each exterior angle of a triangle, the two nonadjacent interior angles are its **remote interior angles.** Name the two remote interior angles for each exterior angle of $\triangle SBM$.

Exterior Angle	Two Remote Interior Angles
$\angle SMT$	$\angle MSB$ and $\angle SBM$
$\angle RSB$	$\angle SMB$ and $\angle SBM$
$\angle QBM$	$\angle BSM$ and $\angle SMB$

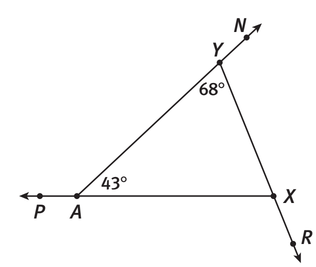
c. Chip claims that there is a relationship between the measure of an exterior angle and its remote interior angles. Examine the measures of the exterior angles and the measures of their corresponding remote interior angles to write a conjecture about their relationship.
The measure of the exterior angle equals the sum of the measures of the two remote interior angles.

Check Your Understanding

3. Determine the value of x.



4. Determine the measure of each of the exterior angles of $\triangle YAX$.



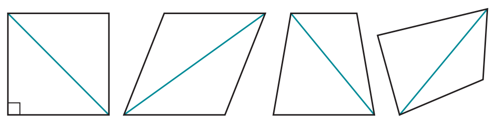
5. The measures of two interior angles of a triangle are 75° and 65°. Determine the measure of the largest exterior angle.

Lesson 17-2
Exterior Angles and Angles in Quadrilaterals

My Notes

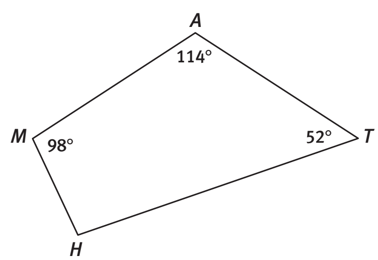
6. Now consider quadrilaterals.

a. Draw a ***diagonal*** from *one* vertex in each quadrilateral.
Sample drawings shown.

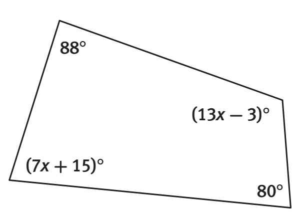


b. Construct viable arguments. What is the sum of the measures of the interior angles in any quadrilateral? Explain your reasoning.
360°. Sample justification: Each of the quadrilaterals is divided into two triangles and the sum of the measures of the angles of a triangle is 180°.

7. Find the unknown angle measure in quadrilateral *MATH*.
96°



8. Determine the value of x in the quadrilateral.
$x = 9$



MATH TERMS

A **diagonal** of a polygon is a line segment connecting two nonconsecutive vertices.

ACTIVITY 17 Continued

6–8 Create Representations, Look for a Pattern, Think-Pair-Share, Discussion Groups In Item 6, students may choose to draw their diagonals from any vertex. They should recognize that they cannot draw a diagonal between two consecutive vertices. No matter where they draw the diagonals, they will divide each quadrilateral into two triangles.

To find the sum of the measures of the interior angles in any convex polygon, find the number of triangles into which the figure can be divided by drawing all possible diagonals from one vertex. For a quadrilateral, students should connect their knowledge of the angle measures in one triangle to their drawings that show two triangles in each quadrilateral. This will help them see that there is a total of 2(180°), or 360°, in the measures of the interior angles of a quadrilateral.

Check Your Understanding

Debrief students' answers to the Check Your Understanding problems to ensure that they can apply the fact that the sum of the measures of the angles in a quadrilateral is 360°.

Answers

9. $x = 23$

10. All of the angles measure 90° (they are right angles). The quadrilateral is a rectangle.

ASSESS

Use the Lesson Practice to assess students' understanding of the exterior angles of a triangle and finding angle measures in quadrilaterals.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

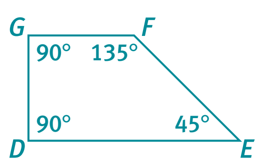
LESSON 17-2 PRACTICE

11. $y = 9$; $m\angle S = 52°$; $m\angle T = 86°$

12. Since $\angle DEB$ is an exterior angle of $\triangle BEF$, $m\angle DEB = 69° + 52° = 121°$. So $m\angle AEB = 121° - 52° = 69°$.

13. 37°

14. Sample sketch:



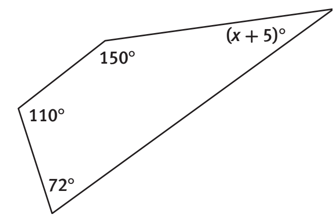
15. No; the measure of the exterior angle must equal the sum of the measures of the two remote interior angles. This means the measure of the exterior angle cannot equal the measure of just one of the remote interior angles.

ADAPT

At this point, students should be able to solve problems related to exterior angles of a triangle and angle measures in a quadrilateral. If students need additional practice working with these concepts, provide some triangles and quadrilaterals and have students measure angles with a protractor to verify the relationships. Additionally, you may wish to assign problems from the Activity Practice.

Check Your Understanding

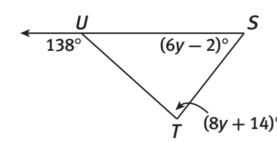
9. Determine the value of x.



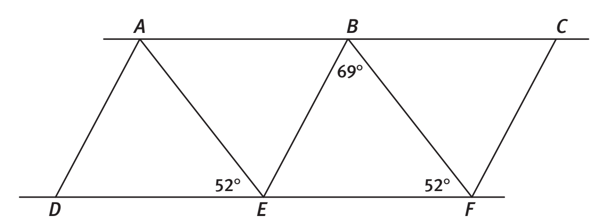
10. In quadrilateral $RSTU$, all of the angles of the quadrilateral are congruent. What can you conclude about the angles? What can you conclude about the quadrilateral?

LESSON 17-2 PRACTICE

11. Determine the value of y. Then find $m\angle S$ and $m\angle T$.



12. A portion of a truss bridge is shown in the figure. Explain how to determine the measure of $\angle AEB$.



13. A quadrilateral contains angles that measure 47°, 102°, and 174°. What is the measure of the fourth angle of the quadrilateral?

14. In quadrilateral $DEFG$, $\angle D$ is a right angle. The measure of $\angle E$ is half the measure of $\angle D$. The measure of $\angle F$ is three times the measure of $\angle E$. Sketch the quadrilateral and label the measure of each angle.

15. **Make use of structure.** Can an exterior angle of a triangle ever be congruent to one of its remote interior angles? Justify your answer.

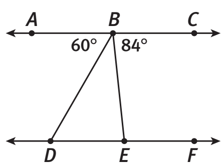
ACTIVITY 17 PRACTICE

Write your answers on notebook paper. Show your work.

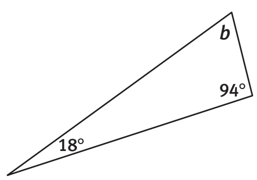
Lesson 17-1

1. Two angles of a triangle measure 32° and 70°. Find the measure of the third angle.

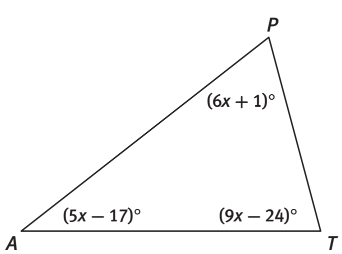
2. In the diagram below, $\overleftrightarrow{AC} \parallel \overleftrightarrow{DF}$. Determine the measure of each of the angles in $\triangle BDE$.



3. Determine the value of b.



4. Write an equation, solve for x, and determine the measure of each angle in $\triangle PAT$.



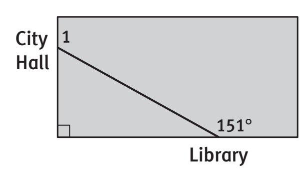
5. The measures of the three interior angles of a triangle are 85°, 20°, and 75°. Determine the measures of the three exterior angles.

6. The measures of two angles of a triangle are 38° and 47°. What is the measure of the third angle?
   A. 85° **B.** 95°
   C. 133° **D.** 142°

7. In $\triangle DEF$, the measure of $\angle D = (3x - 6)°$, the measure of $\angle E = (3x - 6)°$, and the measure of $\angle F = (2x)°$. Which of the following is the measure of $\angle F$?
   A. 24° **B.** 46°
   C. 48° **D.** 66°

8. In $\triangle PQR$, $\angle P$ is an obtuse angle. Which of the following statements about the triangle must be true?
   A. The other two angles must be congruent.
   B. The other two angles must be acute angles.
   C. One of the other two angles could be a right angle.
   D. One of the other two angles could also be an obtuse angle.

9. The figure shows a rectangular lawn at a civic center. Over time, people have cut across the lawn to walk from the library to city hall and made a straight path in the lawn, as shown. What is the measure of $\angle 1$ in the figure?

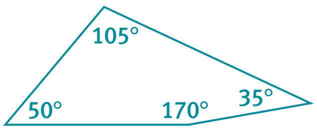


ACTIVITY 17 Continued

ACTIVITY PRACTICE

1. 78°
2. $m\angle BDE = 60°$, $m\angle BED = 84°$, $m\angle DBE = 36°$
3. $b = 68°$
4. $6x + 1 + 5x - 17 + 9x - 24 = 180$; $x = 11$; $m\angle P = 67°$, $m\angle A = 38°$, $m\angle T = 75°$
5. 95°, 105°, 160°
6. B
7. C
8. B
9. 119°

10. $x = 50°$
11. $x = 25$
12. $x = 76°$
13. $m\angle D = 80°$, $m\angle E = 130°$, $m\angle F = 115°$, $m\angle G = 35°$
14. $m\angle 1 = m\angle 2 = 115°$; $m\angle 3 = m\angle 4 = 65°$
15. C
16. B
17. Sample sketch:



18. LaToya is correct; $m\angle BAD = 42° + 29° = 71°$, so $m\angle ADE = 66° + 71° = 137°$, and $m\angle 1 = 180° - 137° - 24° = 19°$.

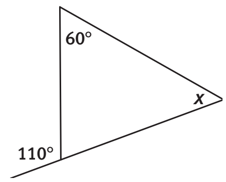
ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.

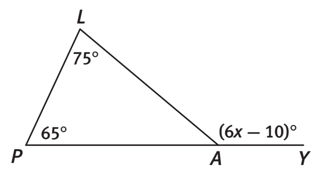
Lesson 17-2

In Items 10–12, determine the value of x.

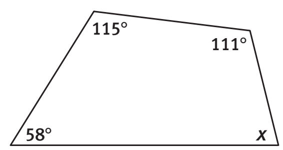
10.



11.

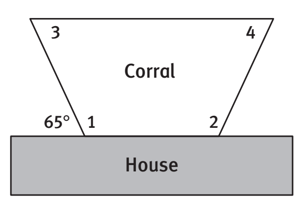


12.



13. Determine the measure of each angle in quadrilateral $DEFG$ with $m\angle D = (12x - 4)°$, $m\angle E = (18x + 4)°$, $m\angle F = (15x + 10)°$, and $m\angle G = (5x)°$.

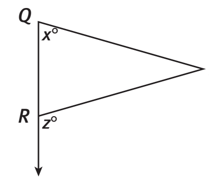
14. The figure shows a plan for a corral in the shape of a trapezoid. One side of the corral is formed by a house and the other three sides are formed by a fence. Given that $\angle 1$ and $\angle 2$ are congruent, and that $\angle 3$ and $\angle 4$ are congruent, find the measures of the four angles.



15. In quadrilateral $ABCD$, $m\angle A = (5x - 5)°$, $m\angle B = (9x)°$, $m\angle C = (12x + 15)°$, and $m\angle D = (15x - 60)°$. Which angle has the greatest measure?

A. $\angle A$ **B.** $\angle B$
C. $\angle C$ **D.** $\angle D$

16. Which expression represents the measure of $\angle P$?

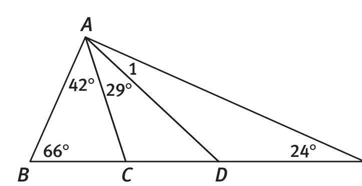


A. $(z + x)°$ **B.** $(z - x)°$
C. $(x - z)°$ **D.** $z°$

17. Sketch a quadrilateral that contains a 50° angle and a 170° angle. Give possible measures for the other two angles.

MATHEMATICAL PRACTICES
Critique the Reasoning of Others

18. Nick and LaToya are painting a backdrop of a mountain for a stage set. A sketch for the backdrop is shown below. Nick says there is not enough information to determine the measure of $\angle 1$. LaToya says there is enough information to determine this angle measure. Who is correct? Explain.

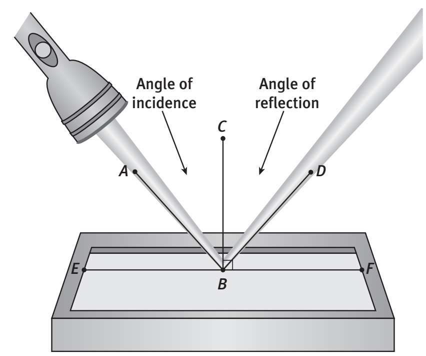


Angle Measures
LIGHT AND GLASS

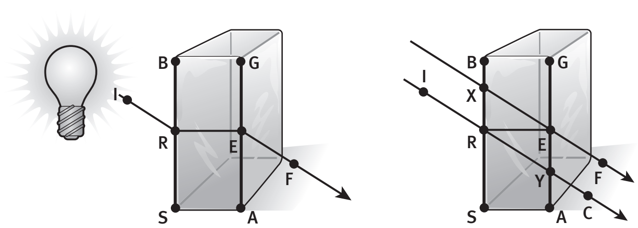
Embedded Assessment 1
Use after Activity 17

A beam of light and a mirror can be used to study the behavior of light. When light hits the mirror it is reflected so that the angle of incidence and the angle of reflection are congruent.

1. Name a pair of nonadjacent complementary angles in the diagram.
2. Name a pair of adjacent supplementary angles in the diagram.
3. In the diagram, $m\angle CBD = (4x)°$ and $m\angle FBD = (3x - 1)°$.
   a. Solve for the value of x.
   b. Determine $m\angle CBD$, $m\angle FBD$, and $m\angle DBE$.



Light rays are bent as they pass through glass. Since a block of glass is a rectangular prism, the opposite sides are parallel and a ray is bent the same amount entering the piece of glass as exiting the glass.



This causes $\overleftrightarrow{XF}$ to be parallel to $\overleftrightarrow{RY}$, as shown.

4. If the measure of $\angle YEX$ is 130°, determine the measure of each of the following angles. Explain how you arrived at your answer.
   a. $\angle BXE$ **b.** $\angle GEF$ **c.** $\angle SRY$
5. If $m\angle CYA = (5x)°$ and $m\angle SRY = (6x - 10)°$, then the value of x is _______.
6. If $m\angle XRE = 90°$ and $m\angle REX = 30°$, then $m\angle RXE =$ _______. Explain how you arrived at your answer.

Common Core State Standards for Embedded Assessment 1

8.G.A.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Embedded Assessment 1

Assessment Focus

- Identify and determine the measures of complementary and supplementary angles
- Determine the measures of the angles of a triangle or quadrilateral
- Determine the measures of the angles formed by parallel lines that are cut by a transversal

Answer Key

1. $\angle EBA$ and $\angle CBD$ or $\angle ABC$ and $\angle DBF$
2. $\angle ABE$ and $\angle ABF$ or $\angle EBC$ and $\angle CBF$ or $\angle EBD$ and $\angle DBF$
3. **a.** $x = 13$
   b. $m\angle CBD = 52°$, $m\angle FBD = 38°$, $m\angle DBE = 142°$
4. **a.** $m\angle BXE = 130°$. Explanations may vary. $\angle BXE$ is alternate interior to $\angle YEX$ and since the lines are parallel they will have the same measure.
   b. $m\angle GEF = 130°$. Explanations may vary. $\angle GEF$ is vertical to $\angle YEX$ so they will have the same measure.
   c. $m\angle SRY = 50°$. Explanations may vary. $\angle BXE$ is corresponding to $\angle XRY$ so they are congruent and $\angle XRY$ is supplementary to $\angle SRY$.
5. $x = 10$
6. $m\angle RXE = 60°$. Explanations may vary. $\angle XRE$, $\angle REX$ and $\angle RXE$ are the three angles of $\triangle REX$. The measures of the angles of a triangle must add up to 180°.

Embedded Assessment 1

7. $x = 51$. The measures of the three angles are 102°, 65°, and 13°.

8. 59°

Teacher to Teacher

You may wish to read through the scoring guide with students and discuss the differences in the expectations at each level. Check that students understand the terms used.

Unpacking Embedded Assessment 2

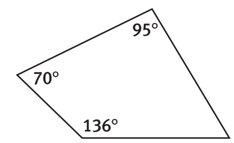
Once students have completed this Embedded Assessment, turn to Embedded Assessment 2 and unpack it with students. Use a graphic organizer to help students understand the concepts they will need to know to be successful on Embedded Assessment 2.

Embedded Assessment 1
Use after Activity 17

Angle Measures
LIGHT AND GLASS

7. The measures of the angles of a triangle are $(2x)°$, $(x + 14)°$, and $(x - 38)°$. Determine the value of x and the measures of each of the three angles.

8. One of the quadrilaterals in a mural design is shown below. Determine the measure of the missing angle.



Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2, 3a-b, 4a-c, 5, 6, 7, 8)	• Clear and accurate understanding of angle relationships, and finding angle measures in a triangle and quadrilateral.	• An understanding of angle relationships and finding angle measures in a triangle and quadrilateral.	• Partial understanding of angle relationships and finding angle measures in a triangle and quadrilateral.	• Little or no understanding of angle relationships and finding angle measures in a triangle and quadrilateral.
Problem Solving (Items 3a-b, 4a-c, 5, 6, 7, 8)	• Interpreting a problem accurately in order to find missing angle measures.	• Interpreting a problem to find missing angle measures.	• Difficulty interpreting a problem to find missing angle measures.	• Incorrect or incomplete interpretation of a problem.
Mathematical Modeling / Representations (Items 1, 2, 3a-b, 4a-c, 5, 6, 7, 8)	• Accurately interpreting figures in order to characterize angle pairs and find angle measures.	• Interpreting figures in order to find angle pairs and find missing angle measures.	• Difficulty interpreting figures in order to find angle pairs and find missing angle measures.	• Incorrectly interpreting figures in order to find angle pairs and find missing angle measures.
Reasoning and Communication (Items 4a-c, 6)	• Precise use of appropriate terms to describe finding angle measures.	• An adequate description of finding of missing angle measures.	• A confusing description of finding missing angle measures.	• An inaccurate description of finding missing angle measures.

Introduction to Transformations

ACTIVITY 18

Move It!

Lesson 18-1 What Is a Transformation?

Learning Targets:

- Recognize rotations, reflections, and translations in physical models.
- Explore rigid transformations of figures.

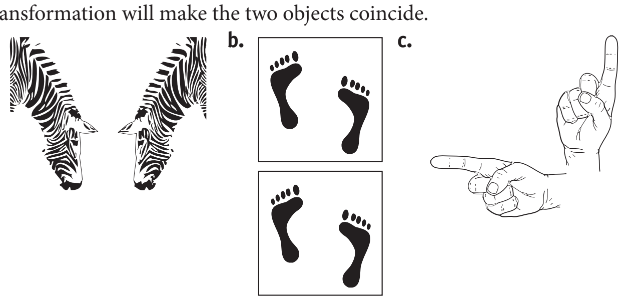
SUGGESTED LEARNING STRATEGIES: Visualization, Create Representations, Vocabulary Organizer, Paraphrasing

A ***transformation***, such as a flip, slide, or turn, changes the position of a figure. Many graphic artists rely on graphic design software to transform images to create logos or promotional materials.

A ***preimage*** is a figure before it has been transformed and the ***image*** is its position after the transformation. You can tell whether a figure has been transformed if the preimage can be moved to coincide with its image.

1. Each set of pictures below shows the preimage and image of some familiar objects. Use the terms *flip*, *slide*, and *turn* to describe what transformation will make the two objects coincide.

a. **b.** **c.**



flip slide turn

2. Make a conjecture about the preimage and image of a transformed object based on your observations of the pictures.

   The original picture, or preimage, and its image are the same size and shape.

My Notes

ACADEMIC VOCABULARY

The word ***transform*** means "to change."

Common Core State Standards for Activity 18

8.G.A.1	Verify experimentally the properties of rotations, reflections, and translations:
8.G.A.1a	Lines are taken to lines, and line segments to line segments of the same length.
8.G.A.1b	Angles are taken to angles of the same measure.
8.G.A.1c	Parallel lines are taken to parallel lines.
8.G.A.3	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

ACTIVITY 18

Investigative

Activity Standards Focus

In Activity 18, students work with transformations, including transformations on the coordinate plane. A key element of this activity is using correct terminology and accurate symbolic representations to describe transformations. In this activity, students work with translations, reflections, and rotations, all of which are rigid motions. Later, in Activity 21, students will be introduced to dilations, which are an example of a non-rigid motion.

Lesson 18-1

PLAN

Pacing: 1 class period

Chunking the Lesson

#1–2 #3–4

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Have students identify a repeating pattern of a shape in the classroom. Then have students write a description of the pattern to explain how the pattern can be generated from a single repeating element. Have students share their descriptions, calling attention to their use of words such as *slide, flip,* and *turn,* as appropriate.

1–2 Visualization, Vocabulary Organizer, Paraphrasing, Interactive Word Wall Students should be able to visualize the relationships in each picture fairly quickly. This item may be used as an assessment of students' ability to recognize basic transformations. Students should immediately start using the correct mathematical vocabulary for each transformation.

Developing Math Language

This lesson introduces several important terms, including *transformation, image,* and *pre-image*. As needed, pronounce new terms clearly and monitor students' pronunciation of terms in their class discussions. Use the class Word Wall to keep new terms in front of students. Include pronunciation guides as needed. Encourage students to review the Word Wall regularly and to monitor their own understanding and use of new terms in their group discussions.

3–4 Visualization, Create Representations, Vocabulary Organizer, Graphic Organizer Item 3 introduces the proper names for the three transformations students saw in Item 1. Explain to students that they will have a chance to work with each transformation in detail in the remaining lessons of the activity. For now, as the terms are discussed, students should make note of the meaning of the line of reflection and the important terminology associated with rotations, including the fixed point. Students can use highlighters or colored pencils to outline these elements in the given diagrams.

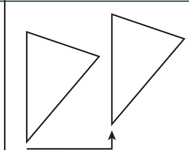
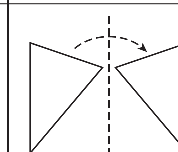
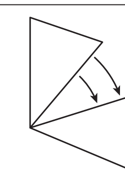
Differentiating Instruction

Support If students have difficulty with Item 4, have them first identify the transformations using the terms *slide*, *flip*, and *turn*. Then have students refer to the table in Item 3 and substitute the correct terminology.

Extend Challenge students to sketch the fixed point in the figures for Item 4c.

My Notes

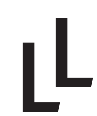
3. **Make use of structure.** The table below shows the proper name for transformations and the corresponding definition. Match each transformation with the words *flip*, *slide*, and *turn*.

Transformation	Definition	Example
Translation slide	Each point of a figure is moved the same distance in the same direction.	
Reflection flip	Each point of a figure is reflected over a line, creating a mirror image.	 line of reflection
Rotation turn	Each point of a figure is rotated through a given angle in a given direction around a fixed point.	

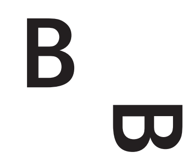
4. For each capital letter shown below, visualize the movement the letter takes while being transformed. Identify the transformation by its proper name.

a. 

reflection

b. 

translation

c. 

rotation

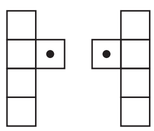
Lesson 18-1
What Is a Transformation?

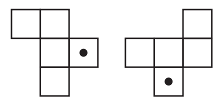
ACTIVITY 18
continued

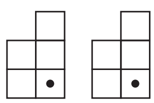
My Notes

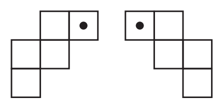
Check Your Understanding

Tell what single transformation, translation, reflection, or rotation will make the figures coincide. Explain how you determined your answers.

5. 

6. 

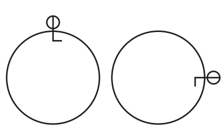
7. 

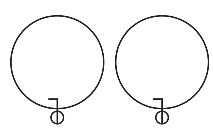
8. 

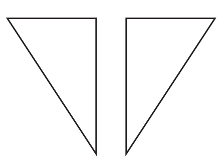
9. Construct viable arguments. How do the sides of the image of a triangle after a translation, reflection, or rotation compare with the corresponding sides of the original figure? How do you know?

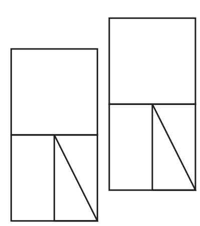
LESSON 18-1 PRACTICE

Each set of figures shows the preimage and image of an object after a single transformation. Describe how the object was transformed using the proper name.

10. 

11. 

12. 

13. 

14. Reason abstractly. Which of the three transformations do you most commonly see in the world around you? Give examples to support your answer.

ACTIVITY 18 Continued

Check Your Understanding

Use these items as a formative assessment of students' ability to recognize and name transformations. Take a few moments to debrief students' responses, asking them to explain how they determined the correct transformation in each case. This will give students additional strategies for identifying transformations. For example, a student may say that the figures in Item 5 are mirror images of each other, so the transformation must be a reflection. Such a response gives all students the clue of "mirror images" for recognizing reflections.

Answers

5. Reflection
6. Rotation
7. Translation
8. Reflection
9. The corresponding sides are equal in length. Possible explanation: You can slide, flip, or turn the original figure onto the image to see that the sides coincide.

ASSESS

Use the Lesson Practice to assess students' understanding of identifying and describing transformations.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 18-1 PRACTICE

10. Rotation
11. Translation
12. Reflection
13. Translation
14. Student answers will vary. Sample answer: Translation is the most common because every time a person moves in some way (walking, running, riding in a car) the person is undergoing a translation.

ADAPT

Check students' work to ensure they are comfortable using correct terminology to describe transformations. Provide cut-outs of different kinds of shapes and have students manipulate them to demonstrate different transformations, tracing around the pre-images and images to record the transformation. Assign problems from the Activity Practice to students who need additional experience identifying and naming transformations.

Lesson 18-2

PLAN

Pacing: 1 class period

Chunking the Lesson

#1 #2–3

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Ask students to write a description of how they can recognize a translation. Then ask volunteers to share their descriptions. Listen for correct mathematical terminology and explain that this lesson will give students additional tools for talking about translations.

1 Shared Reading, Marking the Text, Visualization, Look for a Pattern, Create Representations, Interactive Word Wall You or a student can read aloud while others mark important terms in the text. In addition to recognizing the term *translation*, students should also mark the terms *verbal description, symbolic representation, pre-image* and *image*. If students are struggling to draw the translation image of the triangle, try suggesting a translation of the vertices first. Students may benefit from breaking the ordered pairs into two columns, one for the x-coordinate and one for the y-coordinate, and looking for the pattern in each of the columns.

Developing Math Language

Remind students that they have already seen *symbolic representations* in other situations. For instance, if you buy several notebooks at \$3 each and a pen for \$2, then the expression $3x + 2$ is a symbolic representation of the total cost, with x representing the number of notebooks. You may want to ask students to give additional examples of symbolic representations that they have already seen or used.

My Notes

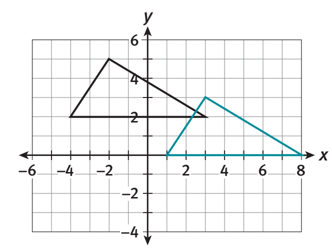
Learning Targets:

- Determine the effect of translations on two-dimensional figures using coordinates.
- Represent and interpret translations involving words, coordinates, and symbols.

SUGGESTED LEARNING STRATEGIES: Visualization, Discussion Groups, Create Representations, Identify a Subtask, Interactive Word Wall

A ***translation*** changes only a figure's position. A verbal description of a translation includes words such as *right, left, up,* and *down*.

1. Consider the triangle shown on the coordinate plane.



Coordinates of Triangle	Coordinates of Image
(−4, 2)	(1, 0)
(−2, 5)	(3, 3)
(3, 2)	(8, 0)

a. Record the coordinates of the vertices of the triangle in the table.
b. Translate the triangle down 2 units and right 5 units. Graph the translation.
c. Record the coordinates of the vertices of the ***image*** in the table.

A ***symbolic representation*** of a transformation is an algebraic way to show the changes to the x- and y-coordinates of the vertices of the original figure, or ***preimage***.

d. **Make use of structure.** Use the information in the table to help you complete the symbolic representation for the translated triangle:
$(x, y) \rightarrow (x + 5, y - 2)$

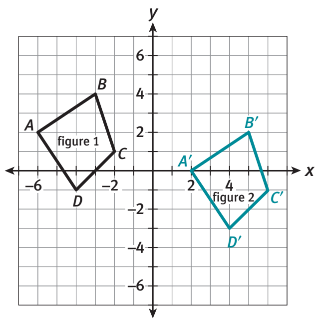
MATH TERMS

A **symbolic representation** of a transformation is an algebraic way to show the changes to the x- and y-coordinates of the vertices of the original figure, or preimage.

A **preimage** is a figure before it has been transformed and the **image** is its position after the transformation.

Lesson 18-2
Translations and Coordinates

2. Figure 2 is the image of Figure 1 after a translation, as shown in the coordinate plane.



a. Record the coordinates of the vertices of the preimage and image.

Figure 1: Preimage		Figure 2: Image	
A	**(−6, 2)**	*A*′	**(2, 0)**
B	**(−3, 4)**	*B*′	**(5, 2)**
C	**(−2, 1)**	*C*′	**(6, −1)**
D	**(−4, −1)**	*D*′	**(4, −3)**

b. Make sense of problems. Refer to the table and graph.
Was the figure translated up or down? **down** By how much? **2 units**

Was the figure translated to the left or right? **right** By how much? **8 units**

c. Write a verbal description to describe the translation.
Figure 1 is translated right 8 units and down 2 units.

d. Describe the translation using a symbolic representation.
symbolic representation: $(x, y) \rightarrow (x + 8, y - 2)$

My Notes

READING MATH

A prime symbol (′) is placed after the letter for the original point to show that the new point is its image.

Example: Point *A*′ is the image of point *A*.

2–3 Visualization, Create Representations, Discussion Groups, Identify a Subtask, Look for a Pattern Begin by discussing the prime symbol as shown in the Reading Math box. Use the diagram on the page to illustrate the notation. In addition, have students mark the image and pre-image on the figure to help them make connections between the terms and the notation.

In order to write the symbolic representation and verbal description, students need to recognize the translation using the figures or the numerical patterns in the table. Some students may find it helpful to write the verbal description before writing the symbolic representation. Others may find it easiest to do this in the opposite order.

Note that Item 3 on the next page is similar to Item 2, but less scaffolding is provided.

Teacher to Teacher

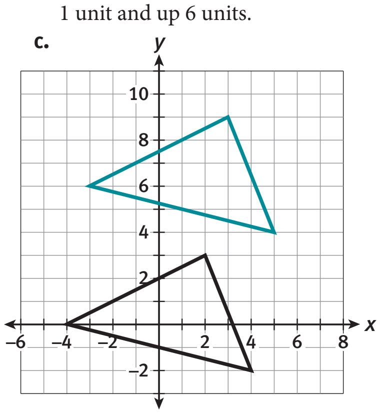
Help students recognize general principles when describing translations. For example, students can recognize that adding or subtracting from the *x*-coordinates moves the figure to the right or left, while adding or subtracting from the *y*-coordinates moves the figure up or down. Also, adding will move the figure in a positive direction, right or up. Subtracting moves the figure in a negative direction, left or down.

Check Your Understanding

Debrief students' answers to these items as a formative assessment of their understanding of translations. Be sure to check that students can write a verbal description and write the corresponding symbolic representation.

Answers

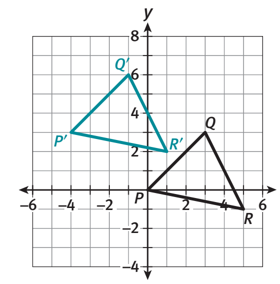
4. a. If a number is added to the x-coordinate, then the figure is translated to the right. If a number is subtracted from the x-coordinate, then the figure is translated to the left. If a number is added to the y-coordinate, then the figure is translated up. If a number is subtracted from the y-coordinate, then the figure is translated down.
   b. The figure is translated right 1 unit and up 6 units.
   c.



5. The number added to or subtracted from the x-coordinate indicates how the point is translated right or left. The number added to or subtracted from the y-coordinate indicates how the point is translated up or down.

My Notes

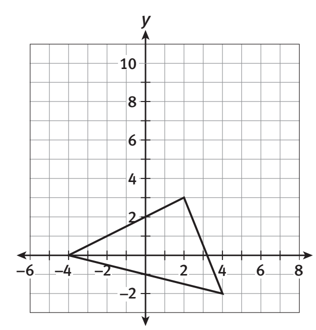
3. The coordinate plane shows $\triangle P'Q'R'$ after $\triangle PQR$ undergoes a translation.



   a. Write a verbal description to describe the translation.
   $\triangle PQR$ is translated left 4 units and up 3 units.
   b. Describe the translation using a symbolic representation.
   symbolic representation: $(x, y) \rightarrow (x - 4, y + 3)$

Check Your Understanding

4. The triangle shown on the coordinate plane is translated according to the following symbolic representation: $(x, y) \rightarrow (x + 1, y + 6)$.



   a. Describe how the symbolic representation can be used to determine if the triangle is translated left or right, and up or down.
   b. Write a verbal description of the translation.
   c. **Attend to precision.** Sketch the image of the triangle according to the symbolic representation.

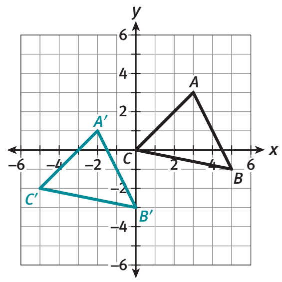
5. **Construct viable arguments.** Explain how the change in the coordinates of a translated point is related to the symbolic representation.

Lesson 18-2
Translations and Coordinates

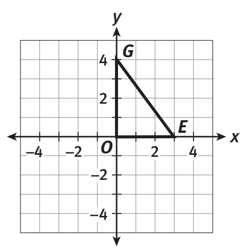
My Notes

LESSON 18-2 PRACTICE

6. Triangle *ABC* is shown along with its image $\triangle A'B'C'$ on the coordinate plane below.
   a. Write a verbal description of the translation.
   b. Show the translation using symbolic representation.



7. Determine the coordinates of the vertices for each image of $\triangle GEO$ after each of the following translations is performed.



   a. 3 units to the left and 3 units down
   b. $(x, y) \rightarrow (x, y - 4)$
   c. $(x, y) \rightarrow (x - 2, y + 1)$
   d. $(x, y) \rightarrow (x - 4, y)$

8. **Critique the reasoning of others.** Quadrilateral *QRST* has vertices $Q(0, 0)$, $R(4, 0)$, $S(4, 4)$, and $T(0, 4)$. Eric states that the image of this quadrilateral after a given translation has vertices $Q'(0, 0)$, $R'(2, 0)$, $S'(2, 2)$, and $T'(0, 2)$. Do you agree or disagree with Eric's statement? Justify your reasoning.

ASSESS

Use the Lesson Practice to assess students' understanding of representing transformations verbally and symbolically.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 18-2 PRACTICE

6. **a.** The figure is translated left 5 units and down 2 units.
   b. Symbolic representation: $(x, y) \rightarrow (x - 5, y - 2)$
7. **a.** $G'(-3, 1)$, $E'(0, -3)$, $O'(-3, -3)$
   b. $G'(0, 0)$, $E'(3, -4)$, $O'(0, -4)$
   c. $G'(-2, 5)$, $E'(1, 1)$, $O'(-2, 1)$
   d. $G'(-4, 4)$, $E'(-1, 0)$, $O'(-4, 0)$
8. No, I do not agree. The pre-image and the image do not coincide. The figures are not the same size.

ADAPT

Review students' work to ensure that they can write translations both verbally and symbolically. If students need additional practice, have them draw a triangle on a coordinate plane and copy it onto a piece of tracing paper to help them translate it to a new location on the coordinate plane. Then have students write a verbal description and symbolic representation for the translation.

Lesson 18-3

PLAN

Pacing: 1 class period

Chunking the Lesson

#1 #2
Check Your Understanding
#4–5
Lesson Practice

TEACH

Bell-Ringer Activity

Ask students to write a description of how they can recognize a reflection. Then ask volunteers to share their descriptions. Listen for correct mathematical terminology and explain that this lesson will give students additional tools for talking about reflections.

1 Shared Reading, Marking the Text, Visualization, Look for a Pattern, Create Representations, Interactive Word Wall You or a student can read aloud while others mark important terms in the text. In addition to recognizing the term *reflection*, a discussion of the term *equidistant* will help students reflect without physically folding the graph. To highlight the line of reflection, ask students to use a colored pencil, highlighter, or their pen to darken the x-axis. For each vertex of $\triangle GHI$, students can count the distance to the line of reflection then count the same distance on the opposite side of the line to plot the image point. For example, the distance from G to the x-axis is 2 units, so the distance from the x-axis to G' is also 2 units.

Teacher to Teacher

Kinesthetic learners may benefit from using tracing paper to make a copy of $\triangle GHI$ and the two axes. Then, they can fold the paper along the x-axis to see the reflection image of the triangle.

CONNECT TO AP

Students may intuitively grasp that $y = x + 5$ is a translation of $y = x + 2$, and that $y = -x$ is a reflection of $y = x$, even if they have not been introduced to transformations of functions. Students will be introduced to transformations of functions in their Algebra classes.

My Notes

MATH TERMS

Equidistant means to be the same distance from a given point or line.

CONNECT TO AP

Translations and reflections of figures in the coordinate plane are preparing you to successfully translate and reflect graphs of functions. This is a helpful tool for visualizing and setting up the graphs for many problems you will solve in calculus.

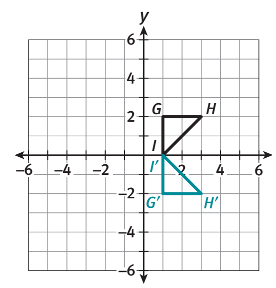
Learning Targets:

- Determine the effect of reflections on two-dimensional figures using coordinates.
- Represent and interpret reflections involving words, coordinates, and symbols.

SUGGESTED LEARNING STRATEGIES: Visualization, Create Representations, Interactive Word Wall, Construct an Argument, Summarizing

To perform a ***reflection***, each point of a preimage is copied on the opposite side of the line of reflection and remains ***equidistant*** from the line.

1. $\triangle GHI$ is shown on the coordinate plane below.



Coordinates of Triangle	Coordinates of Image
G(1, 2)	G′(1, –2)
H(3, 2)	H′(3, –2)
I(1, 0)	I′(1, 0)

a. Record the coordinates of the vertices of $\triangle GHI$ in the table.

b. Sketch the reflection of $\triangle GHI$ over the x-axis.

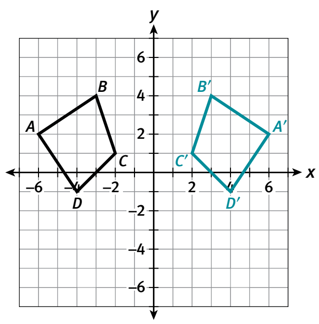
c. Record the coordinates of the vertices of the image $\triangle G'H'I'$ in the table.

The symbolic representation for this transformation is $(x, y) \rightarrow (x, -y)$.

d. Explain how the change in the coordinates of the vertices is related to the symbolic representation for this transformation.

When an object is reflected over the x-axis, the x-coordinates of the image stay the same as those of the preimage, and the y-coordinates change signs.

2. Figure 2 is the image of figure 1 after a reflection, as shown in the coordinate plane.



Preimage: Figure 1		Image: Figure 2	
A	(−6, 2)	*A′*	(6, 2)
B	(−3, 4)	*B′*	(3, 4)
C	(−2, 1)	*C′*	(2, 1)
D	(−4, −1)	*D′*	(4, −1)

a. Record the coordinates of the vertices of the preimage and image.

b. The line across which an object is reflected is called the **line of reflection**. Identify the line of reflection in the transformation of Figure 1.

The line of reflection is the *y*-axis.

c. A verbal description of a reflection includes the line of reflection. Write a verbal description of the reflection.

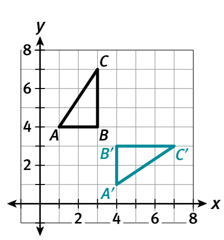
Figure 1 is reflected over the *y*-axis.

d. Describe the reflection using a symbolic representation.

Symbolic Representation: $(x, y) \rightarrow (-x, y)$

Check Your Understanding

3. Triangle *ABC* and its reflected image are shown on the coordinate plane.



Coordinates of △*ABC*		Coordinates of △*A′B′C′*	
A		*A′*	
B		*B′*	
C		*C′*	

a. Complete the table.

b. Identify the line of reflection. Write a verbal description of the transformation.

c. Describe the reflection using symbolic representation.

2 Visualization, Create Representations, Discussion Group, Debriefing Visual learners can easily recognize the *y*-axis as the line of reflection. Other students might benefit from drawing a line segment from a pre-image point to its image point to help them identify the line of reflection.

As students discuss the symbolic representations in Item 2d, point out that the negative sign in $(-x, y)$ is equivalent to multiplying the *x*-coordinate by -1. Be alert for students who may incorrectly think that the value of the coordinate must be negative.

Monitor group discussions to ensure that all members are participating. Pair or group the students carefully to facilitate discussion and understanding of both routine language and mathematical terms.

Check Your Understanding

Debrief students' answers to this item as a formative assessment of their understanding of reflections. Be sure to check that students can write a verbal description and write the corresponding symbolic representation.

Answers

3. a.

Coordinates of △*ABC*		Coordinates of △*A′B′C′*	
A	(1, 4)	*A′*	(4, 1)
B	(3, 4)	*B′*	(4, 3)
C	(3, 7)	*C′*	(7, 3)

b. The line of reflection is $y = x$. Triangle *ABC* is reflected over the line $y = x$.

c. $(x, y) \rightarrow (y, x)$

4. When a figure is reflected over the y-axis, the y-coordinates of the figure stays the same.
5. When a figure is reflected over the x-axis, the x-coordinates of the figure stays the same.

ASSESS

Use the Lesson Practice to assess students' understanding of representing reflections verbally and symbolically.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 18-3 PRACTICE

6. a. Triangle BAM is reflected over the y-axis.

b. Symbolic representation: $(x, y) \rightarrow (-x, y)$

7. a. Keep the x-coordinates the same, and change the sign on the y-coordinates.

b. $C'(-2, -1)$, $D(4, -5)$, and $F(5, -3)$.

8. Sample answer: Filip is not correct. The line of reflection is not the x-axis; it is the line $y = -1$.

ADAPT

Review students' work to ensure that they can write reflections both verbally and symbolically. If students need additional practice, have them draw a triangle on a coordinate plane and copy it onto a piece of tracing paper to help them reflect it across one of the axes. Then have students write a verbal description and symbolic representation for the reflection.

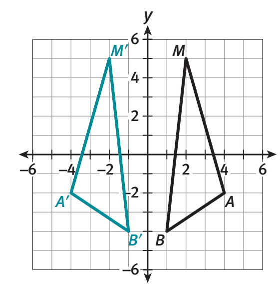
My Notes

4. Express regularity in repeated reasoning. Write a summary statement describing which coordinate stays the same when a figure is reflected over the y-axis.

5. Modify your statement in Item 4 describing which coordinate stays the same when a figure is reflected over the x-axis.

LESSON 18-3 PRACTICE

6. Triangle BAM is shown along with its image $\triangle B'A'M'$ on the coordinate plane below.



a. Write a verbal description of the reflection.

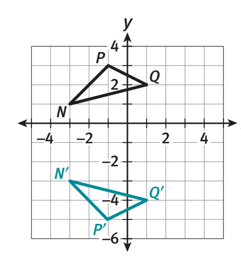
b. Describe the reflection using symbolic representation.

7. Suppose $\triangle CDF$, whose vertices have coordinates $C(-2, 1)$, $D(4, 5)$, and $F(5, 3)$, is reflected over the x-axis.

a. Explain a way to determine the coordinates of the vertices of $\triangle C'D'F'$.

b. Find the coordinates of $\triangle C'D'F'$.

8. Critique the reasoning of others. Filip claims $\triangle N'P'Q'$ is a reflection of $\triangle NPQ$ over the x-axis. Is Filip correct? Justify your answer.



Lesson 18-4
Rotations and Coordinates

Learning Targets:
- Determine the effect of rotations on two-dimensional figures using coordinates.
- Represent and interpret rotations involving words, coordinates, and symbols.

SUGGESTED LEARNING STRATEGIES: Visualization, Create Representations, Look for a Pattern, Interactive Word Wall

A ***rotation*** is a transformation that describes the motion of a figure about a fixed point. To perform a rotation, each point of the preimage travels along a circle the same number of degrees.

1. The point (3, 1) is rotated in a counterclockwise direction about the origin 90°, 180°, and 270°.

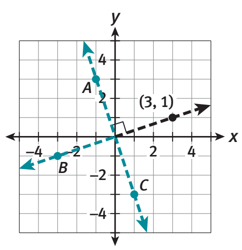


Image Point	Coordinates	Measure of Angle of Rotation
A	**(–1, 3)**	**90°**
B	**(–3, –1)**	**180°**
C	**(1, –3)**	**270°**

a. Write the coordinates of each image point *A*, *B*, and *C* in the table.

b. Complete the table by giving the angle of rotation for each image point.

c. Reason abstractly. Describe in your own words why the origin is the ***center of rotation*** in this rotation transformation.
The preimage point and the image points are all equidistant from the origin.

d. Construct viable arguments. Make a conjecture about the changes of the *x*-and *y*-coordinates when a point is rotated counterclockwise 90°, 180°, and 270° about the origin.
For 90° rotation: Change the sign of the *y*-coordinate of the preimage and then flip the *x*-and *y*-coordinates of the preimage.
For 180° rotation: Change the sign of the *x*-and *y*-coordinates of the preimage.
For 270° rotation: Change the sign of the *x*-coordinate of the preimage and then flip the *x*- and *y*-coordinates of the preimage.

e. What are the coordinates of the point (3,1) after a 360° rotation about the origin? Explain your answer.
(3,1); a 360° rotation is a full turn, so the image point is the same as the preimage.

My Notes

MATH TIP

If the direction of a rotation is counterclockwise, the measure of the angle of rotation is given as a positive value. If the direction of a rotation is clockwise, the measure of the angle of rotation is given as a negative value.

Lesson 18-4

PLAN

Pacing: 1 class period

Chunking the Lesson

#1 #2–4

Check Your Understanding

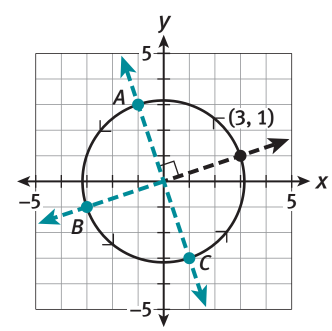
Lesson Practice

TEACH

Bell-Ringer Activity

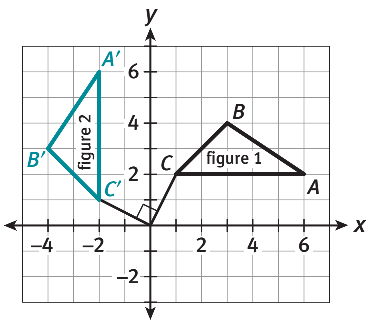
Ask students to write a description of how they can recognize a rotation. Then ask volunteers to share their descriptions. Listen for correct mathematical terminology and explain that this lesson will give students additional tools for talking about rotations.

1 Shared Reading, Marking the Text, Visualization, Look for a Pattern, Create Representations, Interactive Word Wall Have a student read aloud while others mark the text. Remind students what it means to rotate counter-clockwise starting at the pre-image point (3, 1). Have students draw a circle to include all of the points (3, 1), *A*, *B*, and *C*, and then have them indicate the direction of rotation by drawing arrows on the circle, as shown below.



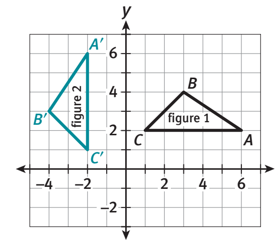
The intent of Item 1 is for students to begin to recognize the pattern in the ordered pairs so they can write the symbolic representations for counter-clockwise rotations.

2–4 Visualization, Create Representations, Discussion Group, Look for a Pattern, Debriefing To visualize the 90° angle of rotation in Item 2, students can draw a segment from (0, 0) to a point and to its image. For example, draw a segment from (0, 0) to C and from (0,0) to C'.



My Notes

2. Figure 2 is a 90° counterclockwise rotation about the origin of figure 1.



Determine the coordinates of the vertices for each figure.

Preimage: Figure 1		Image: Figure 2	
A	**(6, 2)**	A'	**(−2, 6)**
B	**(3, 4)**	B'	**(−4, 3)**
C	**(1, 2)**	C'	**(−2, 1)**

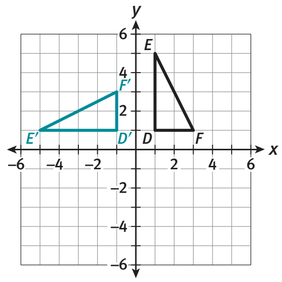
3. Make sense of problems. Complete the summary statement:

When a figure in Quadrant I of the coordinate plane is rotated 90° counterclockwise about the origin, its image is located in Quadrant **II**.

Lesson 18-4
Rotations and Coordinates

My Notes

4. Use appropriate tools strategically. Consider $\triangle DEF$ shown on the coordinate plane.



a. Trace $\triangle DEF$ and the positive x-axis on a piece of tracing paper. Label the vertices and the axis.
b. Rotate the triangle 90° counterclockwise by aligning the origin and rotating the tracing paper until the positive x-axis coincides with the positive y-axis.
c. Record the coordinates of the vertices of the image in the table.

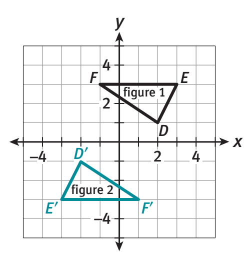
Preimage	$D(1, 1)$	$E(1, 5)$	$F(3, 1)$
Image	$D'(-1, 1)$	$E'(-5, 1)$	$F'(-1, 3)$

d. Sketch $\triangle D'E'F'$ on the coordinate plane above.

Check Your Understanding

5. Make use of structure. Use your results from Items 1, 2, and 3 to write a symbolic representation for a 90° counterclockwise rotation.
$(x, y) \rightarrow (\quad , \quad)$

6. Critique the reasoning of others. Sven recognized the 180° rotation of $\triangle DEF$ about the origin in the coordinate plane and determined the symbolic representation to be $(x, y) \rightarrow (-x, -y)$.



Determine whether the symbolic representation is correct. Justify your answer.

7. A point with coordinates (x, y) is rotated 360° in a counterclockwise direction about the origin. Write the symbolic representation for this transformation:
$(x, y) \rightarrow (\quad , \quad)$.

What does the symbolic representation indicate?

ACTIVITY 18 Continued

2–4 (continued) In Item 4, students are asked to make a copy of $\triangle DEF$ and the positive x-axis on tracing paper. Be sure students draw the x-axis as a ray with endpoint (0, 0). They can place their pencil point on (0, 0) to rotate accurately, keeping the origin in its fixed position on the grid. In order to draw $\triangle D'E'F'$ accurately, students may begin by identifying the coordinates of each ordered pair as the rotated tracing paper rests on the grid.

Check Your Understanding

In Item 5, students summarize their work so far by writing a symbolic representation for a 90° counter-clockwise rotation. Item 6 gives students a chance to apply their knowledge to a new rotation—a 180° counter-clockwise rotation. Be sure to debrief students' responses to these items before having them work on the Lesson Practice.

Answers

5. $(x, y) \rightarrow (-y, x)$
6. The symbolic representation is correct. Sample explanation: $D(2, 1) \rightarrow D'(-2, -1)$, $E(3, 3) \rightarrow E'(-3, -3)$ and $F(-1, 3) \rightarrow F'(1, -3)$. The signs on the x- and y-coordinates are changed.
7. $(x, y) \rightarrow (x, y)$. The coordinates are unchanged after a 360° rotation.

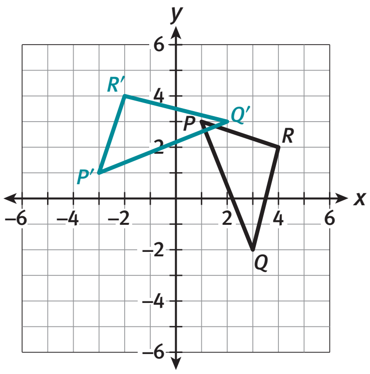
ASSESS

Use the Lesson Practice to assess students' understanding of representing rotations verbally and symbolically.

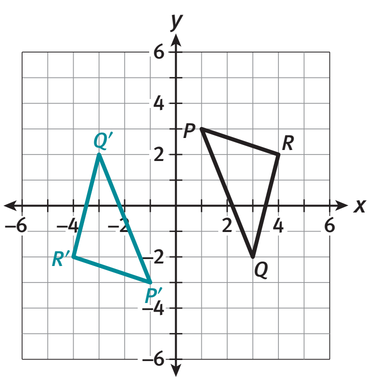
See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 18-4 PRACTICE

8. a. 90° counter-clockwise



b. 180° counter-clockwise



9. (5, 1)
10. a. III
b. IV
11. $(x, y) \rightarrow (y, -x)$
12. Students graphs will vary; however, ordered pairs are related as shown in Item 11.

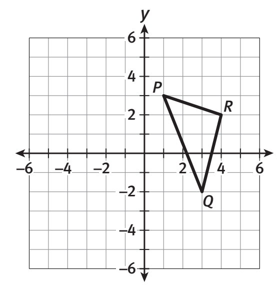
ADAPT

Review students' work to ensure that they can write rotations both verbally and symbolically. If students need additional practice, have them draw a triangle on a coordinate plane and copy it onto a piece of tracing paper to help them rotate it 90° counter-clockwise around the origin. Then have students write a verbal description and symbolic representation for the reflection.

My Notes

LESSON 18-4 PRACTICE

8. Triangle *PQR* with vertices $P(1, 3)$, $Q(3, -2)$, and $R(4, 2)$ is shown on the coordinate plane. Graph each given rotation about the origin.
a. 90° counterclockwise
b. 180° counterclockwise



9. The preimage of point *A* is located at $(-1, 5)$. What are the coordinates of the image, A', after a 270° counterclockwise rotation?

10. Complete the summary statements:
a. When a figure in Quadrant I of the coordinate plane is rotated 180° counterclockwise about the origin, its image is located in Quadrant ______.
b. When a figure in Quadrant I of the coordinate plane is rotated 270° counterclockwise about the origin, its image is located in Quadrant ______.

11. Reason quantitatively. Use your answer from Item 9 to write a conjecture about the symbolic representation for a 270° counterclockwise rotation.

12. Draw a figure on a coordinate plane. Rotate the figure counterclockwise 270° about the origin. How does your drawing confirm your conjecture in Item 11?

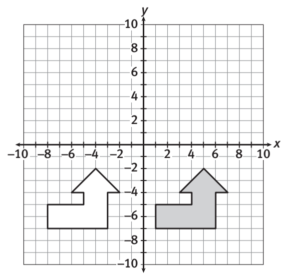
ACTIVITY 18 PRACTICE

Write your answers on notebook paper.
Show your work.

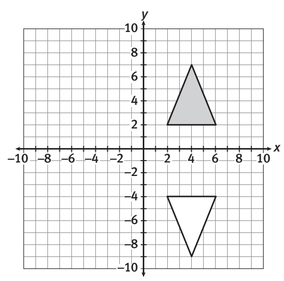
Lesson 18-1

For Items 1–3, the shaded figure is the preimage and the unshaded figure is the image. Identify the single transformation that will make the figures coincide.

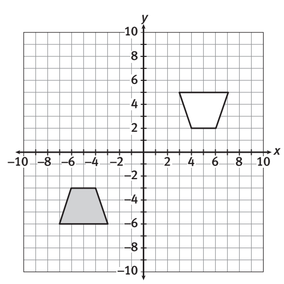
1.



2.

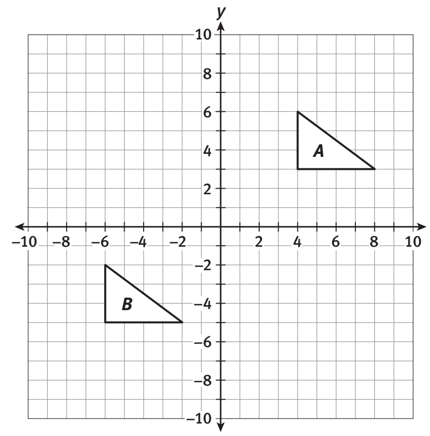


3.



Lesson 18-2

4. Figure *B* is the image of figure *A* after a transformation, as shown in the coordinate plane.

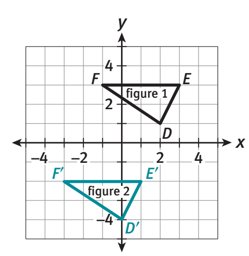


a. Write a verbal description of the transformation.

b. Write a symbolic representation of the transformation.

5. The vertices of $\triangle MOV$ are located at $M(-2, -2)$, $O(4, -2)$, and $V(4, 3)$. Determine the coordinates of the vertices of the image after $\triangle MOV$ is translated 3 units up and 2 units to the right.

6. Which symbolic representation describes the transformation shown on the coordinate plane?



A. $(x, y) \rightarrow (x + 2, y - 5)$
B. $(x, y) \rightarrow (x - 2, y - 5)$
C. $(x, y) \rightarrow (x - 2, y + 5)$
D. $(x, y) \rightarrow (x + 2, y + 5)$

ACTIVITY 18 Continued

ACTIVITY PRACTICE

1. Translation
2. Reflection
3. Rotation
4. **a.** Figure *A* is translated left 10 units and down 8 units.
   b. Symbolic representation: $(x, y) \rightarrow (x - 10, y - 8)$
5. $M'(0, 1)$, $O'(6, 1)$ and $V'(6, 6)$
6. B

7. **a.** $Q'(2, -2)$, $R'(-4, -2)$ and $S'(-4, 4)$
   b. $Q'(-2, 2)$, $R'(4, 2)$, and $S'(4, -4)$
8. **a.** The line of reflection is the x-axis.
   b. Triangle FED is reflected over the x-axis.
   c. Symbolic representation: $(x, y) \rightarrow (x, -y)$
9. **a.** $X'(2, -2)$, $Y'(2, 4)$ and $Z'(-3, 4)$
   b. $X'(2, 2)$, $Y'(-4, 2)$ and $Z'(-4, -3)$
10. **a.** $(-4, -1)$
    b. $(1, -4)$
    c. $(4, 1)$
11. D
12. **a.** $G'(-2, -4)$, $E'(1, 0)$, $O'(-2, 0)$
    b. $G'(-2, -4)$, $E'(1, 0)$, $O'(-2, 0)$
13. Yes; the order matters in some cases. For example, a reflection followed by a 90° rotation generally gives a different image than a 90° rotation followed by a reflection.

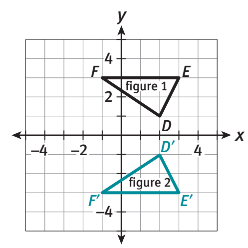
ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.

Lesson 18-3

7. The vertices of $\triangle QRS$ are located at $Q(2, 2)$, $R(-4, 2)$, and $S(-4, -4)$. Determine the coordinates of the vertices of each image of $\triangle QRS$ after the following transformations are performed:
a. $\triangle QRS$ is reflected over the x-axis.
b. $\triangle QRS$ is reflected over the y-axis.

8. Triangle FED and its transformed image is shown on the coordinate plane.



a. Identify the line of reflection.
b. Write a verbal description of the transformation.
c. Write a symbolic representation of the transformation.

Lesson 18-4

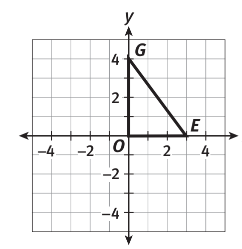
9. The vertices of $\triangle XYZ$ are located at $X(-2, -2)$, $Y(4, -2)$, and $Z(4, 3)$. Determine the coordinates of the vertices of each image of $\triangle XYZ$ after the following transformations are performed:
a. $\triangle XYZ$ is rotated 90° counterclockwise about the origin.
b. $\triangle XYZ$ is rotated 180° about the origin.

10. The preimage of point B is located at $(-1, 4)$. Determine the coordinates of the image, B', for each counterclockwise rotation.
a. 90°
b. 180°
c. 270°

11. Triangle ABC has vertices $A(2, 4)$, $B(5, 7)$, and $C(-1, 5)$. If $\triangle ABC$ is rotated 270° counterclockwise about the origin, in what quadrant(s) would you find the image of $\triangle ABC$?
A. Quadrant I
B. Quadrant III
C. Quadrants II and III
D. Quadrants I and IV

MATHEMATICAL PRACTICES
Make Use of Structure

12. Determine the coordinates of the vertices for each image of $\triangle GEO$ after each of the following transformations is performed.



a. Translate $\triangle GEO$ 2 units to the left and reflect over the x-axis.
b. Reflect $\triangle GEO$ over the x-axis and translate 2 units to the left.

13. Does the order in which multiple transformations, such as rotations, reflections, and translations, are performed on a preimage have an effect on the image?

Rigid Transformations and Compositions

ACTIVITY 19

All the Right Moves

Lesson 19-1 Properties of Transformations

My Notes

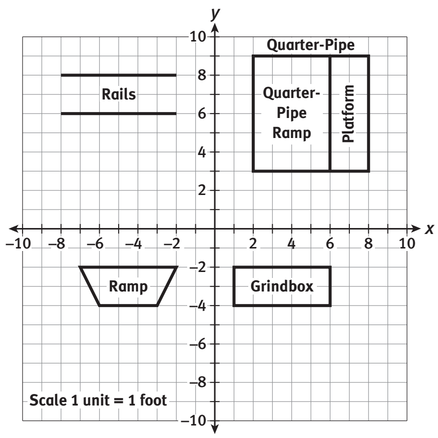
Learning Targets:

- Explore properties of translations, rotations, and reflections on two-dimensional figures.
- Explore congruency of transformed figures.

SUGGESTED LEARNING STRATEGIES: Visualization, Identify a Subtask, Create Representations, Critique Reasoning, Predict and Confirm

Skip and Kate are designing a skateboard park for their neighborhood. They want to include rails, a grindbox, a quarter-pipe, and a ramp. They are deciding where to place the equipment. Kate sketches her plan for the layout on a coordinate plane.

Using the origin as the center of their park, Kate sketched figures to represent the equipment on the coordinate plane, as shown below.



Kate uses the layout on the coordinate plane to determine the dimensions and the area of each figure.

1. **Model with mathematics.** Use the scale on Kate's layout to complete the table of dimensions for each piece of equipment.

Equipment	Base (ft)	Height (ft)	Area (ft^2)
Quarter-Pipe (ramp and platform)	6 ft	6 ft	36 ft^2
Ramp	base 1: 5 ft base 2: 3 ft	2 ft	8 ft^2
Grindbox	5 ft	2 ft	10 ft^2

MATH TIP

The coordinates of the origin on a coordinate plane are (0, 0).

MATH TIP

The area of a trapezoid can be found using the formula

Area $= \frac{1}{2}h(b_1 + b_2)$, where h is the height and b_1 and b_2 are the bases.

Common Core State Standards for Activity 19

8.G.A.1 Verify experimentally the properties of rotations, reflections, and translations:

8.G.A.1a Lines are taken to lines, and line segments to line segments of the same length.

8.G.A.1b Angles are taken to angles of the same measure.

8.G.A.1c Parallel lines are taken to parallel lines.

8.G.A.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

ACTIVITY 19

Investigative

Activity Standards Focus

Students have already been introduced to basic notation and terminology for transformations. In this activity, students investigate properties of transformations and explore the connection between congruence and translations, reflections, and rotations. Students also work with compositions of transformations in this activity.

Lesson 19-1

PLAN

Pacing: 2 class periods

Chunking the Lesson

#1 #2 #3 #4–5

Check Your Understanding

#10–11

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Ask students to find an example of a translation, a reflection, and a rotation in the classroom. Encourage students to consider tile patterns, closet doors, windows, and the positions of desks. Have students share their responses, reminding them to use accurate mathematical terminology.

1 Visualization, Summarizing, Activating Prior Knowledge, Sharing and Responding, Think-Pair-Share

After students have read the introduction, ask a student to summarize the situation. Bring attention to the scale given on the grid. Students will be using the scale on the coordinate grid to find the dimensions and areas of the various pieces of equipment. This item can serve as a formative assessment regarding students' knowledge of area. Students should share their answers with the group and discuss how they found them. If necessary, spend a few minutes reviewing area formulas.

2 Visualization, Identify a Subtask, Create Representations, Critique Reasoning, Predict and Confirm, Debriefing In Item 2, students will reflect the original shape across the *y*-axis and label it with the correct coordinates. The segment separating the ramp and platform should be included in students' drawings; however, students will consider the entire square when determining area and congruence. Students should share answers to this item to hear the ideas of others. Emphasize in student discussions that the dimensions of the pre-image and image are the same, the areas are the same, the corresponding angles are all congruent, and the shape remained the same.

ELL Support

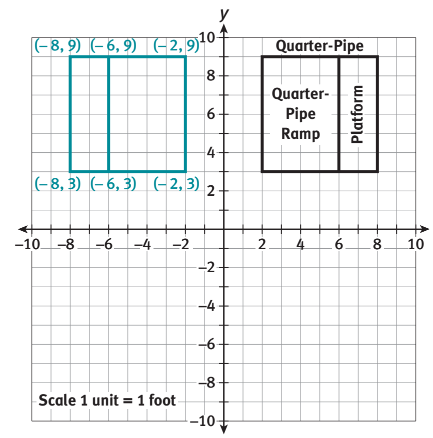
Support Kinesthetic learners may benefit from using tracing paper to help them perform the transformations in this lesson.

Extend Ask students to determine whether the perimeter of each shape remains the same under the various transformations.

My Notes

Skip reviewed Kate's plan for the skateboarding park. To improve the layout, Skip suggested transformations for each piece of equipment as described.

2. The original placement of the quarter-pipe is shown on the coordinate plane.



a. Reflect the figure representing the quarter-pipe ramp and platform over the *y*-axis. Label each vertex of the image with an ordered pair.

b. Determine the dimensions of the image, in feet.

6 feet by 6 feet

c. Compare the areas of the original figure and the image.

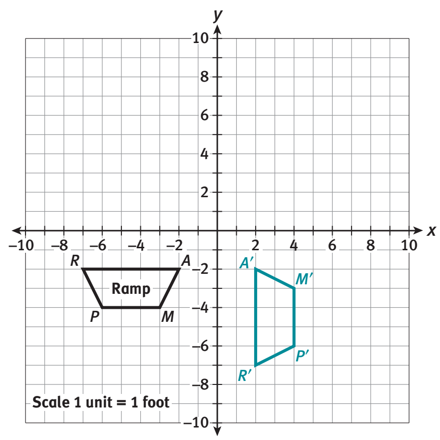
The areas of the two figures are the same: 36 ft^2.

d. Explain why the image is congruent to the original figure.

Sample answer: The figures are congruent because each side of the image is the same size as the original and the image has the same area as the original. The shape and size do not change during a reflection, just the location.

Lesson 19-1
Properties of Transformations

3. The original placement of the ramp is shown on the coordinate plane.



a. Rotate the figure representing the ramp 90° counterclockwise about the origin. Label the vertices of the image R', A', M', and P'.

b. Critique the reasoning of others. Kate states that this rotation will change the shape and size of the figure. Skip reassures her that the image is congruent to the original figure. With whom do you agree? Justify your reasoning.

Skip. Sample answers: The size and shape of a figure do not change during a rotation; after a rotation, the two figures are still trapezoids with bases of the same length and the same side lengths, so the figures are congruent.

Congruent figures have corresponding angles as well as corresponding sides.

c. List the pairs of corresponding angles in trapezoids *RAMP* and $R'A'M'P'$.

angles *R* and *R′*; angles *A* and *A′*; angles *M* and *M′*; angles *P* and *P′*

d. Construct viable arguments. Make a conjecture about the corresponding angles of congruent figures.

The corresponding angles of congruent figures are congruent.

My Notes

3 Visualization, Identify a Subtask, Create Representations, Critique Reasoning, Predict and Confirm, Debriefing Remind students about the meaning of rotating in a counter-clockwise direction. Some students may need to use tracing paper to help them rotate the trapezoid. Students should be showing familiarity with the key ideas of congruence at this point. In particular, the question in Part c is intended to help students recognize that angles are mapped to angles of the same measure after a translation, rotation, or reflection.

Debrief students' work on this item. Monitor presentations to ensure that students are using appropriate words, such as *congruent*, clearly explaining how they applied a concept to a possible solution, and providing justification for why their solution is reasonable. Remind students to use transitions to help them communicate how one thought moves into another.

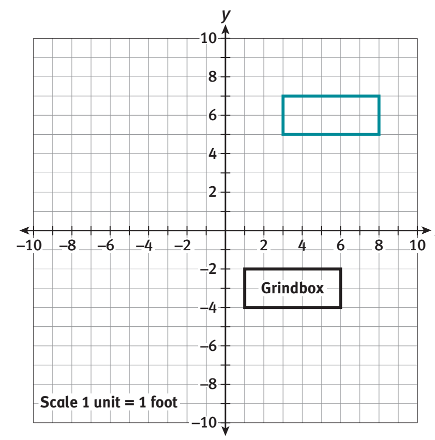
Teacher to Teacher

This lesson introduces the term *congruent figures.* Congruent figures have the same size and shape. If two figures are congruent, one can be obtained from the other through a sequence of translations, reflections, and/or rotations. Also, when two figures are congruent, their corresponding angles are congruent and their corresponding sides are congruent.

4–5 Visualization, Identify a Subtask, Create Representations, Critique Reasoning, Predict and Confirm, Debriefing Item 4 reviews symbolic notation and verbal descriptions of translations. The hallmarks of congruence should be well established at this point. Have students share answers if needed as a formative assessment. In Item 5, students should infer that the original figure and its image are congruent under all of these rigid transformations. By way of contrast, you might mention that there are transformations in which the original figure and its image are not congruent, but only similar. Students will learn about these transformations in a later activity.

My Notes

4. The original placement of the grindbox is shown on the coordinate plane.



a. Plot the image of this figure using the transformation whose symbolic representation is $(x, y) \rightarrow (x + 2, y + 9)$.

b. Write a verbal description of the transformation.

The figure is translated 2 units to the right and 9 units up.

c. Is the image of the grindbox congruent to the preimage of the grindbox? Justify your answer.

Yes. The figures are congruent because they are both rectangles and each side of the image is the same size as the original and the image has the same area as the original. The shape and size do not change during a translation, just the location.

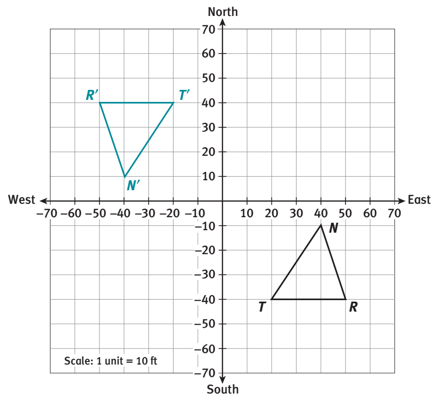
5. Reason abstractly. After using reflections, rotations, and translations to create images of figures, what can you infer about the preimage and its image under all of these transformations?

The image after a translation, reflection, or rotation is congruent to the original figure.

My Notes

Check Your Understanding

Consider $\triangle NTR$ shown on the coordinate grid.

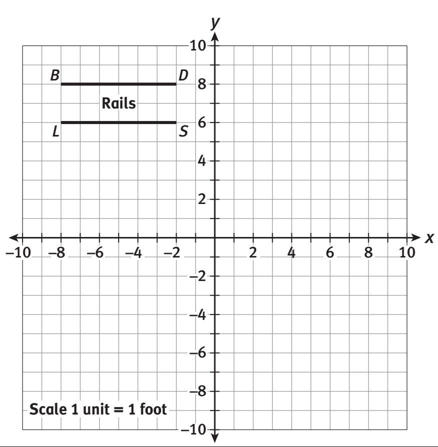


6. Rotate $\triangle NTR$ 180° about the origin. Label the vertices T', R', and N'.
7. Find the area, in square units, of $\triangle NTR$ and $\triangle N'T'R'$. Show the calculations that led to your answer.
8. Write a supporting statement justifying how you know that $\triangle NTR$ and $\triangle N'T'R'$ are congruent triangles.
9. **Express regularity in repeated reasoning.** Could your statement in Item 8 be used to support other types of transformations of $\triangle NTR$? Explain.

MATH TIP

The area of a triangle can be found using the formula
Area $= \frac{1}{2} \times$ base $\times$ height.

Finally, Skip decides to move the location of the rails. The original placement of the rails is shown on the coordinate plane.



ACTIVITY 19 Continued

Check Your Understanding

Debrief students' answers to these items as a formative assessment to check that students understand the connection between congruence and rotations, translations, and reflections.

Answers

6. See student page.
7. Area of $\triangle NTR = \frac{1}{2} \times 3 \times 3 = 4.5$ square units; Area of $\triangle N'T'R'$ $= \frac{1}{2} \times 3 \times 3 = 4.5$ square units
8. Sample answer: $\triangle NTR$ and $\triangle N'T'R'$ are congruent triangles because corresponding sides are congruent and corresponding angles are congruent.
9. Yes; a transformation, such as a rotation, translation, or reflection, does not change the size or shape of the pre-image.

10–11 Visualization, Create Representations, Critique Reasoning, Predict and Confirm, Debriefing, Group Presentation You or a student can read aloud while others mark important terms in the text. Some students may need to be reminded of what it means for line segments to be parallel. In Item 10, students have free choice in moving the rails to any location using each of the three types of rigid transformations: reflection, rotation, and translation. Students should perform the transformations they choose, label the images correctly, and verify that the line segments remain parallel. It is very important to have students share their responses to this question so that a variety of transformations can be seen.

Developing Math Language

Students are introduced to the idea of a *composition of transformations* in this lesson. As you guide students through their learning of this essential mathematical term, explain meanings in terms that are accessible for your students. As much as possible, provide concrete examples to help students gain understanding. Encourage students to make notes about new terms and their understanding of what they mean and how to use them to describe precise mathematical concepts and processes.

My Notes

MATH TERMS

Performing two or more transformations on a figure is called a **composition of transformations**.

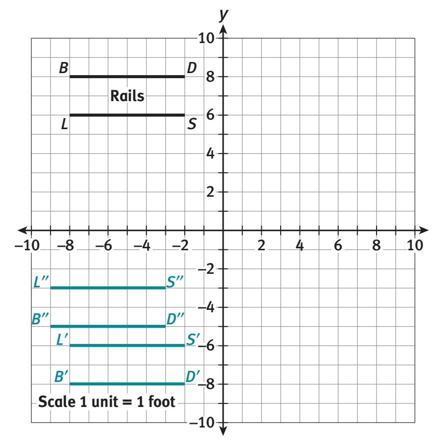
READING MATH

A prime symbol (′) is placed after the letter of the original point to show that the new point is its image. Two prime symbols (″) are placed after the letter of the original point to show that the new point has been transformed twice.

10. Construct viable arguments. Skip claims that the rails are parallel and that moving them, using a reflection, rotation, or translation, will not affect this relationship. Confirm or contradict Skip's claim. Use examples to justify your answer.

Sample answer: The rails will remain parallel. Students should show several transformations on their coordinate planes to justify their answer.

11. Skip decides to move the rails using a ***composition of transformations***.



a. Reflect the graph of each rail, $\overline{BD}$ and $\overline{LS}$, over the x-axis. Label the image points B', D', L', and S'.

b. Then, translate the reflected image 3 feet up and 1 foot left. Label the image points B'', D'', L'', and S''.

My Notes

Check Your Understanding

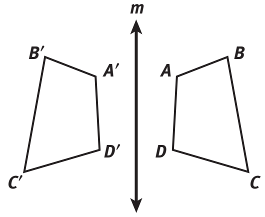
12. Refer to Item 11. Describe a method to determine if $\overline{B''D''}$ and $\overline{L''S''}$ are congruent to $\overline{BD}$ and $\overline{LS}$.

13. Describe how the rails in Item 11 would differ in orientation if the translation in Item 11b was changed to a counterclockwise rotation 90° about the origin.

14. Do you agree with the statement that congruency is preserved under a composition of transformations involving translations, reflections, and rotations? If not, provide a counterexample.

LESSON 19-1 PRACTICE

15. Quadrilateral *ABCD* is reflected across line *m* as shown in the diagram.



a. Name the side that corresponds to $\overline{CD}$ and explain the relationship between the lengths of these two segments.

b. Name the angle that corresponds to angle *C* and explain the relationship between the measures of these two angles.

16. Draw a coordinate plane on grid paper. Create and label a triangle having vertices *D*(3, 5), *H*(0, 8), and *G*(3, 8). Perform each transformation on the coordinate plane.

a. Reflect $\triangle DHG$ across the *x*-axis.

b. Rotate $\triangle DHG$ 90° counterclockwise about the origin.

c. Translate $\triangle DHG$ 4 units right.

d. Which of the transformed images above are congruent to $\triangle DHG$?

ACTIVITY 19 Continued

Check Your Understanding

Use these items to summarize and debrief students' learning in this lesson. Pay attention to students' work and their discussions to be sure they understand that rigid transformations (translations, reflections, and rotations) preserve congruence and parallelism.

Answers

12. Sample answer: Determine the length of each segment and compare.

13. Sample answer: The rails would appear vertical in orientation rather than horizontal.

14. Yes

ASSESS

Use the Lesson Practice to assess students' understanding of transformations and congruence.

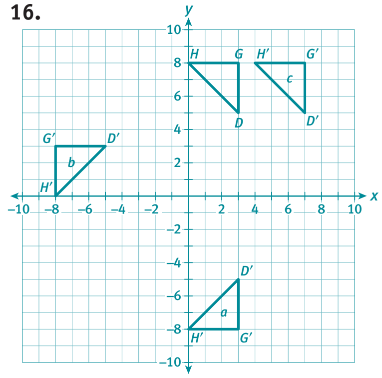
See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 19-1 PRACTICE

15. a. $\overline{CD}$ corresponds to $\overline{C'D'}$; The two segments are congruent.

b. Angle *C* corresponds to angle *C′*; The two angles have the same measure.

16.



d. All of the transformed images are congruent to $\triangle DHG$.

ADAPT

Check students' work to ensure that students can perform transformations, identify corresponding sides and angles, and identify congruent figures that are created under rigid transformations. If students need additional experience with these concepts, assign problems from the Activity Practice.

Lesson 19-2

PLAN

Materials

- scissors

Pacing: 2 class periods

Chunking the Lesson

#1–8 #9

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Give students a few minutes to write an example of the symbolic notation for a translation, a reflection, and a rotation. Ask students to give a verbal description of each transformation. Then have volunteers share their work. Debriefing students' responses will serve as a review of symbolic notation, which will be helpful to students as they play the game that is introduced in this lesson.

Teacher to Teacher

Students check each other's work as part of this game. Make an answer key for each *All the Right Moves* Game Card by copying the answers on page 259. The answer keys will be used to check the coordinates for each position and for the composition of transformations. According to the game's rules, students get 2 points for each correctly identified transformation and 5 points for each set of correct coordinates, for a possible total score of 21 points for each game.

1–8 Visualization, Close Reading, Shared Reading, Marking the Text The game instructions can be read as a whole class. Make sure students clearly understand how the game is played before they begin. Go over the game sheets on pages 255 and 256. Then have students go to page 259 and cut out one game piece for each player and one set of *All the Right Moves* Game Cards for each pair of students. (There is a set of Game Cards in each book, but only one set of cards per pair of students is needed.) Extra copies of the game pieces are also provided.

Lesson 19-2
Composition of Transformations

My Notes

Learning Targets:

- Explore composition of transformations.
- Describe the effect of composition of translations, rotations, and reflections on two-dimensional figures using coordinates.

SUGGESTED LEARNING STRATEGIES: Self Revision/Peer Revision, Visualization, Discussion Groups, Create Representations, Close Reading

To explore composition of transformations, you and a partner will play a game called All the Right Moves. Cut out the five All the Right Moves cards on page 259 and two game pieces. You and your partner will use only one set of All the Right Moves game cards to play the game, but you both need a game piece.

DISCUSSION GROUP TIPS

As you read and discuss the rules of All the Right Moves, ask and answer questions to be sure you have a clear understanding of not only all the terminology used, but also how the game is to be played.

All the Right Moves Rules

1. As partners, lay out the 5 All the Right Moves cards face down.
2. Take turns choosing an All the Right Moves card. You will each take 2 cards. The extra card may be used later as a tiebreaker.
3. Working independently, each of you will use your All the Right Moves cards to complete the two game sheets on pages 255 and 256.
4. To complete the first game sheet, follow these steps:
   a. Record the number of one of your All the Right Moves cards on your game sheet. You may use either one first.
   b. Plot and label the points for Position 0 on the grid. Then use those points as the vertices to draw a triangle.
   c. Follow the directions on the All the Right Moves card to find the coordinates of the vertices for Position 1.
   d. Record the new coordinates on your game sheet, plot the new points on the coordinate plane, and draw a triangle. Use your game piece to identify the transformation you made and record its name on your game sheet.
   e. Continue until you have moved the figure to all 5 positions on the All the Right Moves card. Then record the coordinates of the composition of transformations, which is Position 5.
5. Repeat the process with your other All the Right Moves card for the second game sheet.
6. When you and your partner have completed your two cards, exchange game sheets and check each other's work.
7. Score your game sheets: You get 2 points for each transformation you correctly identify and 5 points for the correct coordinates of each composition of transformations.
8. The player with the greater number of points wins the game.

Teacher to Teacher

To aid students in using their game piece, have them turn it over and label the vertices on the back of the piece. When the piece is reflected, students will be able to see the letter that names each vertex.

ACTIVITY 19
continued

My Notes

All the Right Moves
Game Sheet

Player: ______________________________

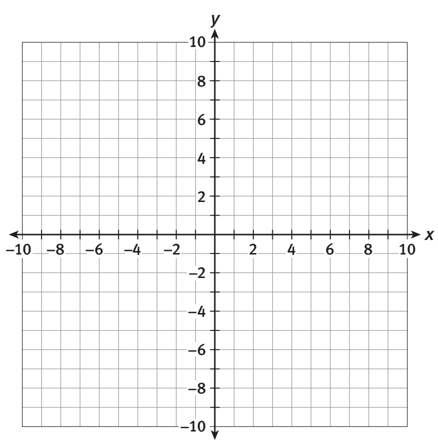
All the Right Moves **Card:** ______________________________

Position 0: A( ), B( ), C( )	**Type of Transformation**
Position 1: A( ), B( ), C( )	
Position 2: A( ), B( ), C( )	
Position 3: A( ), B( ), C( )	
Position 4: A( ), B( ), C( )	
Position 5: A( ), B( ), C( )	

Composition of Transformations:

A(   ), ***B***(   ), ***C***(   )

Points Earned for All the Right Moves **Card:** ______________



ACTIVITY 19
continued

My Notes

All the Right Moves
Game Sheet

Player: ______________________________

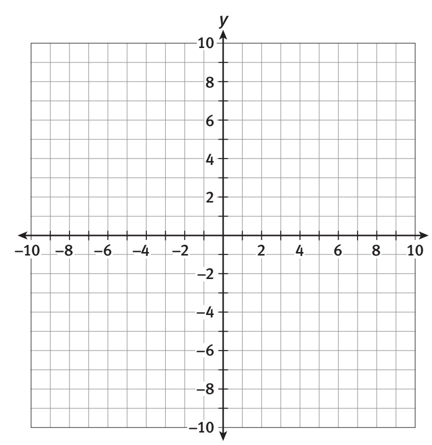
All the Right Moves **Card:** ______________________________

Position 0: *A*( ), *B*( ), *C*( )	**Type of Transformation**
Position 1: *A*( ), *B*( ), *C*( )	
Position 2: *A*( ), *B*( ), *C*( )	
Position 3: *A*( ), *B*( ), *C*( )	
Position 4: *A*( ), *B*( ), *C*( )	
Position 5: *A*( ), *B*( ), *C*( )	

Composition of Transformations:

A(   ), ***B***(   ), ***C***(   )

Points Earned for All the Right Moves **Card:** ______________



Total Points Earned: ______________

Lesson 19-2
Composition of Transformations

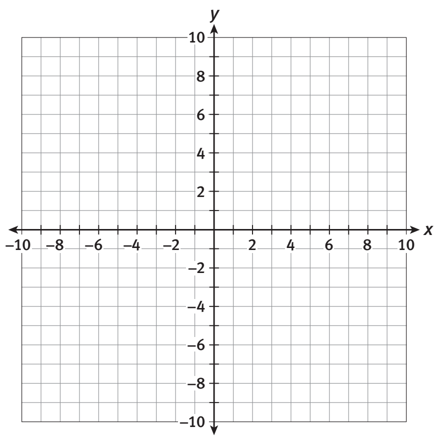
My Notes

9. Model with mathematics. Work with your partner to discover a composition of transformations that has the same result as one from the All the Right Moves game but takes fewer transformations.

As students check each other's work, they will see more ways in which various combinations of transformations will yield the same result.

a. Select one of the All the Right Moves game cards.

b. Follow the instructions on the card and use the coordinate plane below to draw the locations of Position 0 and Position 5.



c. Use what you know about reflections, translations, and rotations to move the game piece from Position 0 to Position 5 in four or fewer steps.

d. Write the directions for the moves you found in Item 9c on a separate sheet of paper. Then trade directions with your partner and follow each other's directions to see whether the new transformation is correct.

9 Self Revision/Peer Revision, Discussion Groups, Debriefing, Sharing and Responding, Think-Pair-Share Now students are asked to use the same *All the Right Moves* game cards, but to find a shorter way to get from Position 0 to Position 5. They start by drawing the original shape and final image on their grids, then look for a different sequence of transformations between the two figures. Using the game pieces will help students test ideas about possible moves. Students should then write the new directions, and exchange their answers with a partner as a way of checking their work. This item can also serve as informal assessment regarding students' understanding of transformations. Be sure to debrief this item so that students can hear a range of ideas. Students are likely to see a variety of compositions of transformations with differing levels of complexity.

Check Your Understanding

These items serve as a formative assessment of students' understanding of compositions of transformations. Check students' work to ensure that they can perform compositions of transformations on the coordinate plane and can identify the coordinates of the image of a figure after a series of transformations.

Answers

10. $T' = (5, 1)$; $T'' = (-5, 1)$

11. a. $A'(3, -2)$, $B'(-2, -8)$, $C'(-5, -1)$

b. $A'(7, -2)$, $B'(2, -8)$, $C'(-1, -1)$

ASSESS

Use the Lesson Practice to assess students' understanding of compositions of transformations.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 19-2 PRACTICE

12. (3, 1)

13. $(x, y) \rightarrow (x - 1, y + 4)$

14. $(x, y) \rightarrow (-x, -y)$

15. a. Sample answer: First, reflect the figure over the y-axis; then, reflect the figure over the x-axis; then translate the figure three units to the right and three units up.

b. Sample answer: Rotate the figure counterclockwise 90° and then translate the figure to the right one unit and down one unit.

ADAPT

These problems require students to perform and analyze compositions of transformations. Check students' work to determine if they need additional practice with these concepts. If so, assign problems from the Activity Practice. You may also want to have students draw their own pairs of congruent triangles on a coordinate plane and then describe a sequence of transformations that maps one triangle to the other. Encourage students to find multiple compositions that work.

Check Your Understanding

10. The point $T(5, -1)$ is reflected across the x-axis, then across the y-axis. What are the coordinates of T' and T''?

11. $\triangle ABC$ has vertices $A(-5, 2)$, $B(0, -4)$, and $C(3, 3)$.

a. Determine the coordinates of the image of $\triangle ABC$ after a translation 2 units right and 4 units down followed by a reflection over the y-axis.

b. What are the coordinates of the image of $\triangle ABC$ after a reflection over the y-axis followed by a translation 2 units right and 4 units down?

LESSON 19-2 PRACTICE

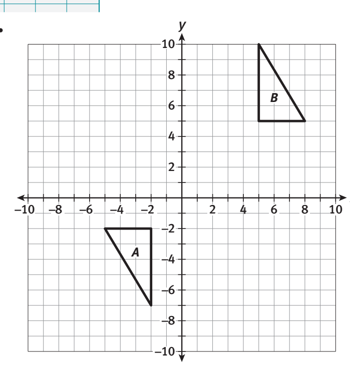
12. The point (1, 3) is rotated 90° about the origin and then reflected across the y-axis. What are the coordinates of the image?

13. Attend to precision. Find a single transformation that has the same effect as the composition of translations $(x, y) \rightarrow (x - 2, y + 1)$ followed by $(x, y) \rightarrow (x + 1, y + 3)$. Use at least three ordered pairs to confirm your answer.

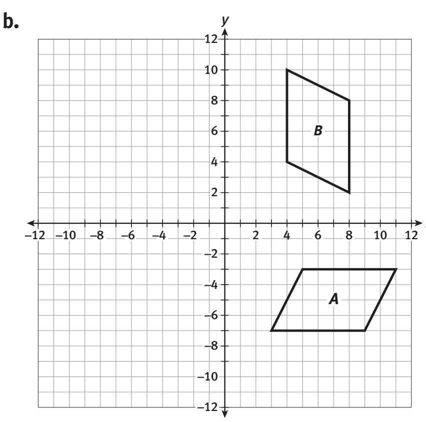
14. Reason abstractly. Describe a single transformation that has the same effect as the composition of transformations reflecting over the x-axis followed by reflecting over the y-axis. Use at least three ordered pairs to confirm your answer.

15. Write a composition of transformations that moves figure A so that it coincides with figure B.

a.

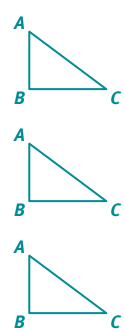


b.



Lesson 19-2
Composition of Transformations

All the Right Moves Game Cards

All the Right Moves Card 1	All the Right Moves Card 2	All the Right Moves Card 3
Position 0: $A(3, 4)$, $B(3, 1)$, $C(7, 1)$ **Position 1:** $(x, y) \rightarrow (-x, y)$ **Position 2:** $(x, y) \rightarrow (x + 3, y + 4)$ **Position 3:** $(x, y) \rightarrow (x, -y)$ **Position 4:** $(x, y) \rightarrow (x - 1, y)$ **Position 5:** $(x, y) \rightarrow (-y, x)$	**Position 0:** $A(1, 1)$, $B(1, -2)$, $C(5, -2)$ **Position 1:** $(x, y) \rightarrow (x + 2, y)$ **Position 2:** $(x, y) \rightarrow (-x, y)$ **Position 3:** $(x, y) \rightarrow (x, 4 - y)$ **Position 4:** $(x, y) \rightarrow (-y, x)$ **Position 5:** $(x, y) \rightarrow (x - 1, y)$	**Position 0:** $A(-5, 0)$, $B(-5, -3)$, $C(-1, -3)$ **Position 1:** $(x, y) \rightarrow (-y, x)$ **Position 2:** $(x, y) \rightarrow (-x, y)$ **Position 3:** $(x, y) \rightarrow (-6 - x, y)$ **Position 4:** $(x, y) \rightarrow (x + 3, y + 4)$ **Position 5:** $(x, y) \rightarrow (-y, x)$
All the Right Moves Card 4	**All the Right Moves Card 5**	**Game Pieces**
Position 0: $A(-3, -4)$, $B(-3, -7)$, $C(1, -7)$ **Position 1:** $(x, y) \rightarrow (-x, y)$ **Position 2:** $(x, y) \rightarrow (x + 5, y)$ **Position 3:** $(x, y) \rightarrow (x + 2, y - 1)$ **Position 4:** $(x, y) \rightarrow (4 - x, y)$ **Position 5:** $(x, y) \rightarrow (y, -x)$	**Position 0:** $A(0, -1)$, $B(0, -4)$, $C(4, -4)$ **Position 1:** $(x, y) \rightarrow (y, -x)$ **Position 2:** $(x, y) \rightarrow (x, 2 - y)$ **Position 3:** $(x, y) \rightarrow (x, -y)$ **Position 4:** $(x, y) \rightarrow (x + 2, y)$ **Position 5:** $(x, y) \rightarrow (-x, -y)$	Cut out one game piece for each partner. 

My Notes

ACTIVITY 19 Continued

Game Card Answers

Card 1

Position 0: $A(3, 4)$, $B(3, 1)$, $C(7, 1)$	Type of Transformation
Position 1: $A(-3, 4)$, $B(-3, 1)$, $C(-7, 1)$	reflection
Position 2: $A(0, 8)$, $B(0, 5)$, $C(-4, 5)$	translation
Position 3: $A(0, -8)$, $B(0, -5)$, $C(-4, -5)$	reflection
Position 4: $A(-1, -8)$, $B(-1, -5)$, $C(-5, -5)$	translation
Position 5: $A(8, -1)$, $B(5, -1)$, $C(5, -5)$	rotation

Composition of Transformations:
$A(8, -1)$, $B(5, -1)$, $C(5, -5)$

Card 2

Position 0: $A(1, 1)$, $B(1, -2)$, $C(5, -2)$	Type of Transformation
Position 1: $A(3, 1)$, $B(3, -2)$, $C(7, -2)$	translation
Position 2: $A(-3, 1)$, $B(-3, -2)$, $C(-7, -2)$	reflection
Position 3: $A(-3, 3)$, $B(-3, 6)$, $C(-7, 6)$	reflection
Position 4: $A(-3, -3)$, $B(-6, -3)$, $C(-6, -7)$	rotation
Position 5: $A(-4, -3)$, $B(-7, -3)$, $C(-7, -7)$	translation

Composition of Transformations:
$A(-4, -3)$, $B(-7, -3)$, $C(-7, -7)$

Card 3

Position 0: $A(-5, 0)$, $B(-5, -3)$, $C(-1, -3)$	**Type of Transformation**
Position 1: $A(0, -5)$, $B(3, -5)$, $C(3, -1)$	rotation
Position 2: $A(0, -5)$, $B(-3, -5)$, $C(-3, -1)$	reflection
Position 3: $A(-6, -5)$, $B(-3, -5)$, $C(-3, -1)$	reflection
Position 4: $A(-3, -1)$, $B(0, -1)$, $C(0, 3)$	translation
Position 5: $A(1, -3)$, $B(1, 0)$, $C(-3, 0)$	rotation

Composition of Transformations:

$A(1, -3)$, $B(1, 0)$, $C(-3, 0)$

Card 4

Position 0: $A(-3, -4)$, $B(-3, -7)$, $C(1, -7)$	**Type of Transformation**
Position 1: $A(3, -4)$, $B(3, -7)$, $C(-1, -7)$	reflection
Position 2: $A(8, -4)$, $B(8, 7)$, $C(4, -7)$	translation
Position 3: $A(10, -5)$, $B(10, 6)$, $C(6, -8)$	translation
Position 4: $A(-6, -5)$, $B(-6, 6)$, $C(-2, -8)$	reflection
Position 5: $A(-5, 6)$, $B(6, 6)$, $C(-8, 2)$	rotation

Composition of Transformations:
$A(-5, 6)$, $B(6, 6)$, $C(-8, 2)$

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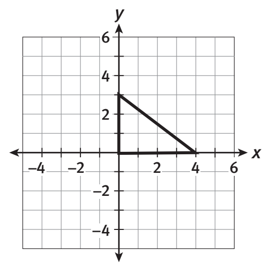
ACTIVITY 19 PRACTICE

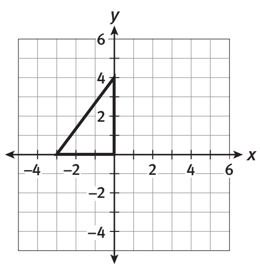
Write your answers on notebook paper. Show your work.

Lesson 19-1

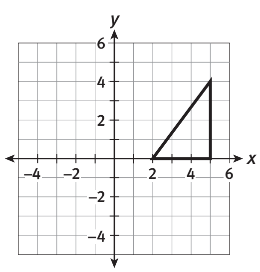
Each figure in Items 1–4 is an image of the figure shown on the coordinate plane below. Describe the transformations that were performed to obtain each image.



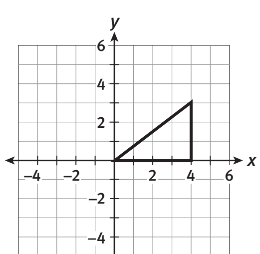
1.



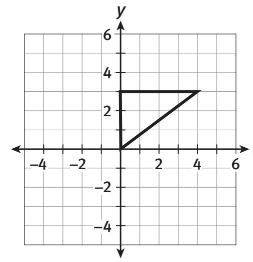
2.



3.



4.



5. Compare the figures in Items 1–4.

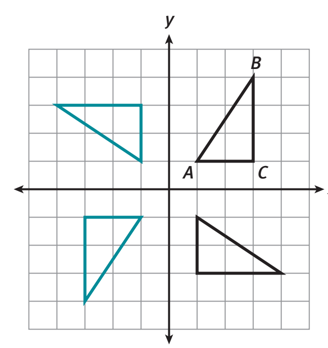
a. How do the areas of each figure compare to the area of the original figure?

b. What can you determine about the corresponding sides of each figure?

c. What can you determine about the corresponding angles of each figure?

d. Can you determine if the images of each figure are congruent to the original figure? Provide reasoning for your answer.

6. The coordinate plane below shows $\triangle ABC$ and a 90° clockwise rotation of $\triangle ABC$ about the origin.



a. Sketch the 180° clockwise rotation of $\triangle ABC$.

b. Sketch the 90° counterclockwise rotation of $\triangle ABC$.

c. How do the images compare with $\triangle ABC$?

ACTIVITY 19 Continued

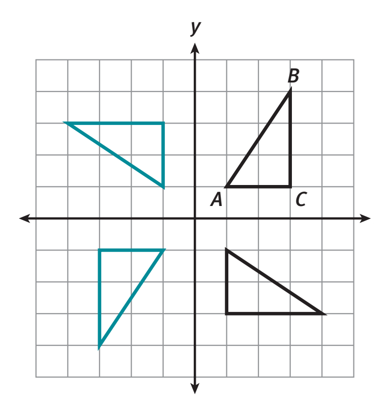
Card 5

	Type of Transformation
Position 0: $A(0, -1)$, $B(0, -4)$, $C(4, -4)$	
Position 1: $A(-1, 0)$, $B(-4, 0)$, $C(-4, -4)$	rotation
Position 2: $A(-1, 2)$, $B(-4, 2)$, $C(-4, 6)$	reflection
Position 3: $A(-1, -2)$, $B(-4, -2)$, $C(-4, -6)$	reflection
Position 4: $A(1, -2)$, $B(-2, -2)$, $C(-2, -6)$	translation
Position 5: $A(-1, 2)$, $B(2, 2)$, $C(2, 6)$	rotation

Composition of Transformations:
$A(-1, 2)$, $B(2, 2)$, $C(2, 6)$

ACTIVITY PRACTICE

1. Rotation
2. Translation then rotation
3. Translation then reflection
4. Reflection then rotation
5. a. The areas are the same.
b. Corresponding sides are congruent.
c. Corresponding angles are congruent.
d. Yes; Sample answer: The figures are all congruent because corresponding sides are congruent and corresponding angles are congruent.
6. a–b.



c. They are congruent.

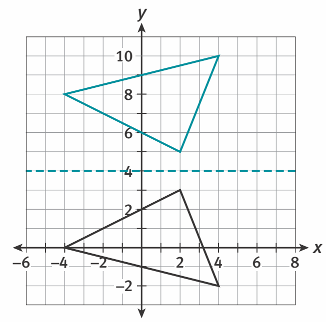
7. D
8. No; transformations, such as a rotation, translation, or reflection, does not change the size or shape of the pre-image no matter what order the transformations are performed.
9. C
10. Answers will vary.
11. $(x, y) \rightarrow (x + 4, y)$
12. A horizontal reflection and a vertical reflection combined will give the same result as a 180° rotation.

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.

Lesson 19-2

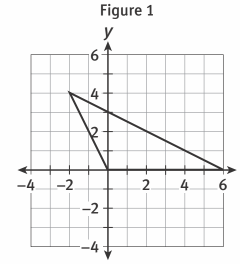
7. The preimage of a triangle is shown on the coordinate plane.

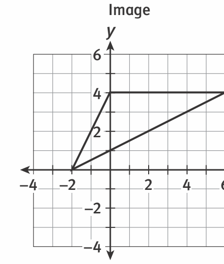


Which of the following types of transformation results in an image where corresponding angles and sides are NOT congruent?
A. reflection
B. rotation
C. translation
D. none of the above

8. To create a logo, Henry transforms a quadrilateral by reflecting it over the x-axis, translating it 4 units up and then rotating the image 270° counterclockwise about the origin. Does the order in which Henry performs the transformations on the preimage change the size or shape of the image? Explain.

9. Figure 1 shows the preimage of a figure.





Which of the following transformation(s) have been performed on Figure 1 to obtain the image?
A. Rotate 180°.
B. Shift down 2 units and reflect over the line $y = 2$.
C. Reflect over the x-axis and shift up 4 units.
D. Reflect over the y-axis and shift up 4 units.

10. List two transformations and then name one transformation that gives the same result as the two transformations.

11. Find a translation that has the same effect as the composition of translations $(x, y) \rightarrow (x + 7, y - 2)$ followed by $(x, y) \rightarrow (x - 3, y + 2)$.

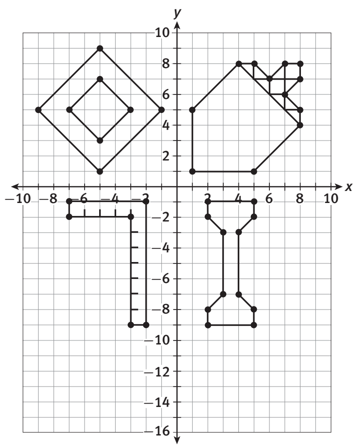
MATHEMATICAL PRACTICES
Reason Abstractly

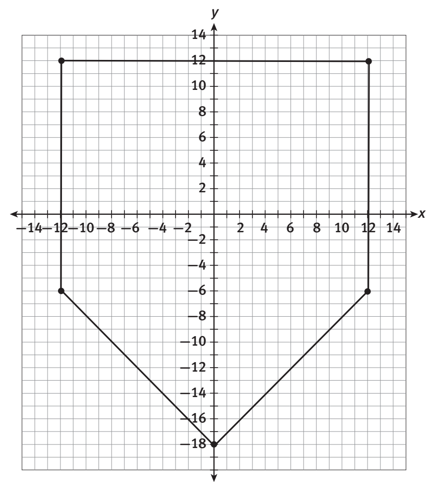
12. How many and what types of reflections would have to be performed on a preimage to get the same image as a 180° rotation?

Rigid Transformations
IN TRANSFORMATIONS WE TRUST

Embedded Assessment 2
Use after Activity 19

In medieval times, a person was rewarded with a coat of arms in recognition of noble acts. In honor of your noble acts so far in this course, you are being rewarded with a coat of arms. Each symbol on the coordinate plane below represents a special meaning in the history of heraldry.





Transform the figures from their original positions to their intended positions on the shield above using the following descriptions.

1. The **acorn** in Quadrant I stands for antiquity and strength and is also the icon used in the SpringBoard logo.
   a. Reflect the acorn over the x-axis. Sketch the image of the acorn on the shield.
   b. Write the symbolic representation of this transformation.
2. The **mascle** in Quadrant II represents the persuasiveness you have exhibited in justifying your answers.
   a. Rotate the mascle 270° counterclockwise about the origin. Sketch the image of the mascle on the shield.
   b. Write the symbolic representation of this transformation.
3. The **carpenter's square** in Quadrant III represents your compliance with the laws of right and equity. The location of the carpenter's square is determined by a composition of transformations. Rotate the carpenter's square 90° counterclockwise about the origin followed by a reflection over the y-axis.
   a. Copy and complete the table by listing the coordinates of the image after the carpenter's square is rotated 90° counterclockwise about the origin.
   b. Sketch the image of the carpenter's square after the composition of transformations described.

Preimage	Image
(−7, −1)	
(−7, −2)	
(−2, −1)	
(−3, −2)	
(−3, −9)	
(−2, −9)	

Common Core State Standards for Embedded Assessment 2

8.G.A.1 Verify experimentally the properties of rotations, reflections, and translations:
8.G.A.1a Lines are taken to lines, and line segments to line segments of the same length.
8.G.A.1b Angles are taken to angles of the same measure.
8.G.A.1c Parallel lines are taken to parallel lines.
8.G.A.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

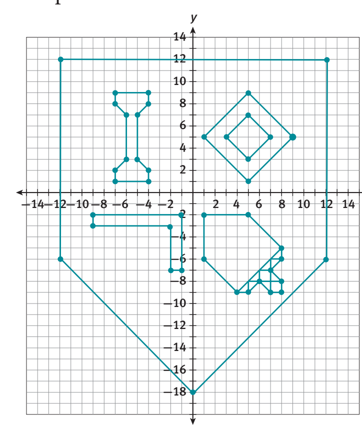
Embedded Assessment 2

Assessment Focus

- Perform translations, reflections, and rotations on the coordinate plane
- Identify transformations that preserve congruence

Answer Key

Grid after all transformations are complete.



1. **a.** See the grid above.
   b. $(x, y) \rightarrow (x, -y)$
2. **a.** See the grid above.
   b. $(x, y) \rightarrow (y, -x)$
3. **a.**

Pre-Image	Image
(−7, −1)	(1, −7)
(−7, −2)	(2, −7)
(−2, −1)	(1, −2)
(−3, −2)	(2, −3)
(−3, −9)	(9, −3)
(−2, −9)	(9, −2)

   b. See the grid above.

4. a. See the grid above.
   b. Left 9, up 10
5. Answers may vary. Each of the transformations is a rigid transformation. The image after a rigid transformation will always be congruent to the pre-image.

Teacher to Teacher

You may wish to read through the scoring guide with students and discuss the differences in the expectations at each level. Check that students understand the terms used.

Unpacking Embedded Assessment 3

Once students have completed this Embedded Assessment, turn to Embedded Assessment 3 and unpack it with students. Use a graphic organizer to help students understand the concepts they will need to know to be successful on Embedded Assessment 3.

Embedded Assessment 2
Use after Activity 19

Rigid Transformations
IN TRANSFORMATIONS WE TRUST

4. Finally, the **column** in Quadrant IV represents the determination and steadiness you've shown throughout your work in this course.
   a. Sketch the column using the transformation given by the symbolic representation $(x, y) \rightarrow (x - 9, y + 10)$.
   b. Write a verbal description of the transformation.
5. Explain why each of the symbols on your coat of arms is congruent to the preimage of the symbol on the original coordinate plane.

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1a-b, 2a-b, 3a-b, 4a-b, 5)	• Clear and accurate understanding of reflections, rotations, and translations in the coordinate plane.	• An understanding of reflections, rotations, and translations in the coordinate plane with few errors.	• Partial understanding of reflections, rotations, and translations in the coordinate plane.	• Incorrect understanding of reflections, rotations, and translations in the coordinate plane.
Problem Solving (Items 1a-b, 2a-b, 3a-b, 4a-b)	• Interpreting a problem accurately in order to carry out a transformation.	• Interpreting a problem to carry out a transformation.	• Difficulty interpreting a problem to carry out a transformation.	• Incorrect or incomplete interpretation of a transformation situation.
Mathematical Modeling / Representations (Items 1a, 2a, 3a, 4a)	• Accurately transforming pre-images and drawing the images.	• Transforming pre-images and drawing the images with few, if any, errors.	• Difficulty transforming pre-images and drawing the images.	• Incorrectly transforming pre-images and drawing the images.
Reasoning and Communication (Items 4b, 5)	• A precise explanation of congruent transformations.	• An understanding of transformations that retain congruence.	• A confusing explanation of congruent transformations.	• An inaccurate explanation of congruent transformations.

Similar Triangles

Mirrors and Shadows

Lesson 20-1 Exploring Similarity

ACTIVITY 20

My Notes

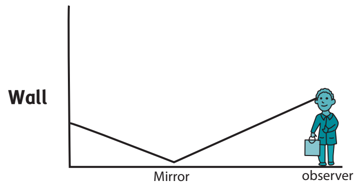
Learning Targets:

- Identify similar triangles.
- Identify corresponding sides and angles in similar triangles.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Visualization, Create Representations, Group Discussion

Thales of Miletus was a Greek philosopher, mathematician, and scientist who lived in 600 B.C.E. Two thousand six hundred years ago, he wondered about the height of the Great Pyramid in Egypt. Thales noticed that the sun's shadows fell from every object in the desert at the same angle, creating similar triangles from every object. Thales's research allowed him to use similar triangles to measure the height of the pyramids of Egypt and the distance to a ship at sea.

Thales used shadows in his work; however, a mirror placed on the floor can also be used to determine measures indirectly. When the mirror is placed at a particular distance from the wall, the distance that an observer stands from the mirror determines the reflection that the observer sees in the mirror.



1. Use the table on the next page to record results for each of the steps below.
- Find a spot on the floor 20 feet away from one of the walls of your classroom.
- Place a mirror on the floor 4 feet from that wall.
- Each group member should take a turn standing on the spot 20 feet from the wall and look into the mirror. Other group members should help the observer locate the point on the wall that the observer sees in the mirror and then measure the height of this point above the floor.
- Before moving the mirror, each group member should take a turn as the observer.
- Repeat the same process by moving the mirror to locations that are 8 feet and 10 feet away from the wall.

Answers will vary depending on eye levels of each student.

Common Core State Standards for Activity 20

8.G.A.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

ACTIVITY 20

Guided

Activity Standards Focus

In this activity, students develop an understanding of similarity, especially as it applies to similar triangles. Students identify corresponding sides and angles in similar triangles, and use properties of the sides and angles of similar triangles to solve real-world and mathematical problems.

Lesson 20-1

PLAN

Materials

- small mirror (one per group)
- tape measure
- sticky notes

Pacing: 2 class periods

Chunking the Lesson

#1 #2–3 #4–6 #7

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Ask students to write several ratios that are proportional. Have students share their ratios and ask them to explain how they know that the ratios are proportional.

Teacher to Teacher

Students are to collect and analyze data based on students' heights. If possible, arrange students into groups whose members are not all the same height.

Before having students measure using feet, it may be necessary to review fractions of a foot. For example: 3 inches $= \frac{1}{4}$ ft, 8 inches $= \frac{2}{3}$ ft, and so on.

1 Shared Reading, Marking the Text, Use Manipulatives The table for Item 1 should be completed by all students in each group, with each student taking the role of a different person (A, B, C, or D). Data will vary depending on students' heights.

A sticky note works well for locating the point on the wall that the observer sees in the mirror. Have students draw a large dot on a sticky note. A group member can slowly move the sticky note up the wall until the observer views it in the mirror. Then the group member can stick the note to the wall to mark the spot for measurement.

2–3 Look for a Pattern, Visualization, Create Representations, Think-Pair-Share To complete Item 3, students may want to convert the mixed numbers in Items 1 and 2 to decimals. Ask students to label the diagrams with the equivalent decimal values before completing the table. Each ratio in Item 3b should be close to $\frac{1}{4}$ or 0.25.

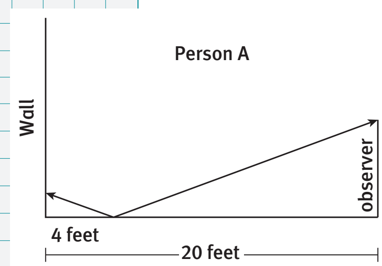
My Notes

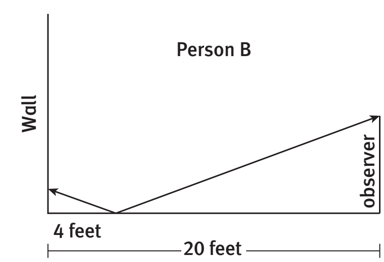
Distance from the Wall to the Mirror (in feet)	Height of the Point on the Wall Reflected in the Mirror (in feet)			
	Person A	Person B	Person C	Person D
4				
8				
10				

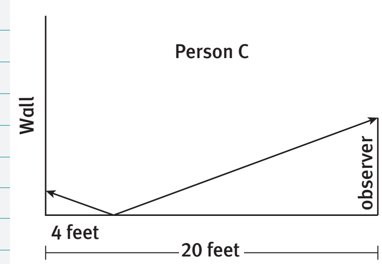
2. Measure the eye-level height for each member of the group and record it in the table below.
Answers will vary depending on eye-level height of each student.

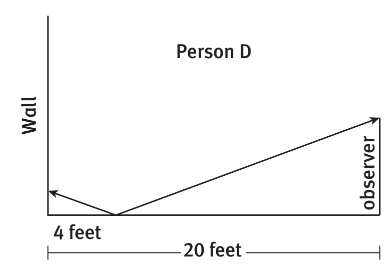
Eye-Level Height for Each Group Member			
Person A	Person B	Person C	Person D

3. Consider the data collected when the mirror was 4 feet from the wall.
a. On the diagrams below, label the height of each group member and the height of the point on the wall determined by the group member.









b. For each person in the group, determine the ratio of the height of the point on the wall to the eye-level height of the observer.
Ratios should be close to 0.25; however, inaccuracies in measurements may cause slight variations.

Ratio of height of the point on the wall to eye level of observer		Person A	Person B	Person C	Person D
	Ratio as a fraction				
	Ratio as a decimal				

c. Express regularity in repeated reasoning. What appears to be true about the ratios you found?
Answers may vary. All ratios are close to 0.25.

4. If the eye-level height of a five-year-old observer is 3.6 feet, what height can you predict for the point on the wall? Explain your reasoning.
0.9 feet; The ratio of the height of the point on the wall to the eye-level height of the observer should be 0.25, so the height of the point on the wall must be 0.9 feet since $0.9 \div 3.6 = 0.25$.

5. Consider the data collected when the mirror was 8 feet from the wall. For each group member, determine the ratio of the height of the point on the wall to the eye-level height of the observer. What appears to be true?
Answers may vary. The ratios are all close to $\frac{2}{3}$ or 0.667.

6. Consider the data collected when the mirror was 10 feet from the wall. For each group member, determine the ratio of the height of the point on the wall to the eye-level height of the observer. What appears to be true?
Answers may vary. The ratios are all close to 1.

ACTIVITY 20 Continued

4–6 Look for a Pattern, Predict and Confirm, Discussion Groups Item 4 can be used to assess student understanding of proportional reasoning. One approach to solving this problem is to use a proportion where y is the desired height for the spot on the wall seen by the five-year-old child. Another approach is for students to simulate the eye-level height of 3.6 feet and measure the height of the spot on the wall directly.

In Items 5 and 6, students determine the ratio of the height of the point on the wall to the eye-level height of the observer using the same process as in Item 3. Allowing for some error in measurements, students should conclude that the ratios in each item appear to be equal. In Item 5 students should recognize the ratio $\frac{2}{3}$ and in Item 6 students should conclude that the ratio is close to 1.

Teacher to Teacher

Students are introduced to the definition of similar triangles and asked to apply this definition to triangles in future items. The term *similar figures* may be added to the Interactive Word Wall as part of the discussion of this text. It is important for students to understand what is meant by *similarity statement*. One similarity statement is given with this explanation; however, other equivalent similarity statements can be written; for example, $\triangle WMP \sim \triangle FME$, $\triangle MPW \sim \triangle MEF$, and $\triangle PMW \sim \triangle EMF$.

Developing Math Language

This lesson introduces the term *similarity*, which has both a mathematical meaning and an everyday meaning. Evaluate students' use of all types of vocabulary in their written responses to ensure that they are using everyday words as well as academic vocabulary and math terms correctly.

Help students to understand that a *proportion* is formed by two equivalent ratios. As needed, pronounce new terms clearly and monitor students' pronunciation of terms in their class discussions. Use the class Word Wall to keep new terms in front of students. Include pronunciation guides as needed. Encourage students to review the Word Wall regularly and to monitor their own understanding and use of new terms in their group discussions.

My Notes

MATH TERMS

Similar polygons are polygons in which the lengths of the corresponding sides are in proportion, and the corresponding angles are congruent.

MATH TERMS

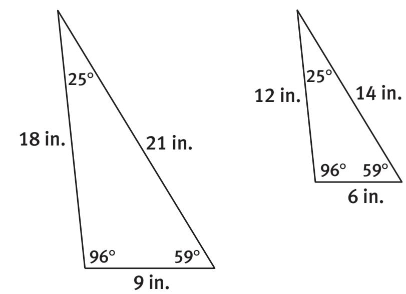
When two ratios are equivalent, then they form a **proportion**. For example, the ratios $\frac{3}{7}$ and $\frac{12}{28}$ are equivalent. Setting these ratios equal generates the proportion $\frac{3}{7} = \frac{12}{28}$.

WRITING MATH

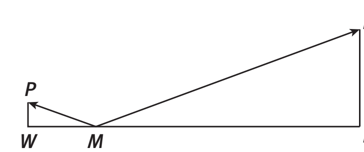
The symbol $\sim$ is used to denote two similar figures.

Similar polygons are polygons in which the lengths of the corresponding sides are in ***proportion***, and the corresponding angles are congruent.

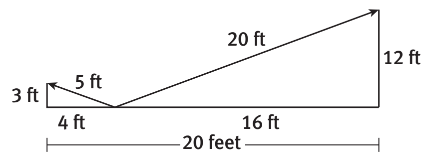
For example, in the following triangles, the corresponding angles are congruent, and the corresponding sides are in proportion. Therefore, the triangles are similar.



A similarity statement for the triangles below is $\triangle PWM \sim \triangle EFM$. A ***similarity statement*** indicates that the corresponding angles are congruent, and the corresponding sides are proportional.



7. The diagram below shows two similar triangles like the triangles you worked with in Item 3.



a. Use the lengths of the three pairs of corresponding sides to create three ratios in the form $\frac{\text{side length in small triangle}}{\text{corresponding length in large triangle}}$

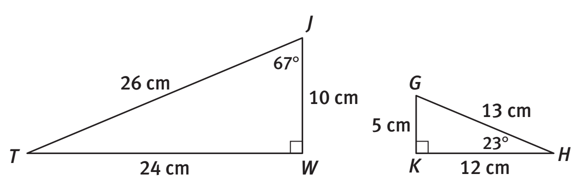
$\frac{3}{12}, \frac{4}{16}, \frac{5}{20}$

b. Compare the ratios written in part a. Then explain how these ratios relate to the ratios you created in Item 3.
All of the ratios in Part a are equal to 0.25, which is close to the ratios found in Item 3.

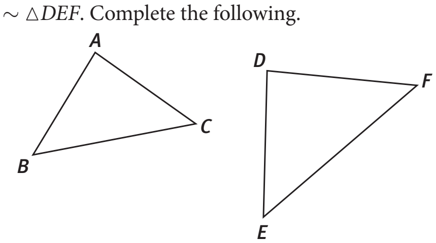
My Notes

Check Your Understanding

8. Are the triangles shown below similar? If so, explain why and write a similarity statement. If not, explain why not.



9. In the figure, $\triangle ABC \sim \triangle DEF$. Complete the following.
a. $m\angle F =$ ______
b. $\frac{AB}{DE} = \frac{\square}{DF}$



ACTIVITY 20 Continued

7 Look for a Pattern, Create Representations, Think-Pair-Share
Students are asked to create ratios using the corresponding sides of two triangles. Stress the importance of correctly identifying corresponding sides. In this item, the 20-ft side (longest side) in the large triangle corresponds to the 5-ft side (longest side) in the small triangle. The 12-ft side (shortest side) in the large triangle corresponds to the 3-ft side (shortest side) in the small triangle. Lastly, the 16-ft side in the large triangle corresponds to the 4-ft side in the small triangle. Students should look for a pattern and realize that all ratios are equal to $\frac{1}{4}$. If necessary, you may elect to lead a class discussion on how to identify equivalent fractions.

Check Your Understanding
Identifying similar triangles is an essential skill for students' future work in geometry. Item 8 serves as a formative assessment of this skill, while Item 9 assesses students' ability to identify corresponding parts of similar triangles.

Answers
8. Yes; corresponding angles are congruent and corresponding sides are proportional. $\triangle JWT \sim \triangle GKH$
9. a. $m\angle C$
   b. AC

ASSESS

Students' answers to lesson practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 20-1 PRACTICE

10. Yes; corresponding angles are congruent and corresponding sides are proportional. $\triangle RLG \sim \triangle NCP$

11. $\triangle DEF$ is similar to $\triangle ABC$ because corresponding angles are congruent and corresponding sides are proportional.

12. $\triangle DFE \sim \triangle ABC$; yes, the statement can be written in other forms, such as $\triangle DEF \sim \triangle ACB$, etc.

13. $\overline{DE}$ and $\overline{AC}$, $\overline{DF}$ and $\overline{AB}$, $\overline{FE}$ and $\overline{BC}$

14. The triangles cannot be similar since corresponding angles are not congruent. One triangle has angles that measure 36°, 51°, and 93°. The other triangle has angles that measure 36°, 49°, and 95°.

ADAPT

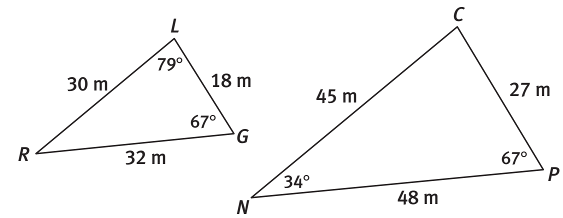
Check students' answers to the Lesson Practice to be sure they can identify similar triangles and identify corresponding parts. Emphasize the definition of similar figures as you discuss students' work and remind students that they will have opportunities to apply similar triangles in the next lesson.

ACTIVITY 20
continued

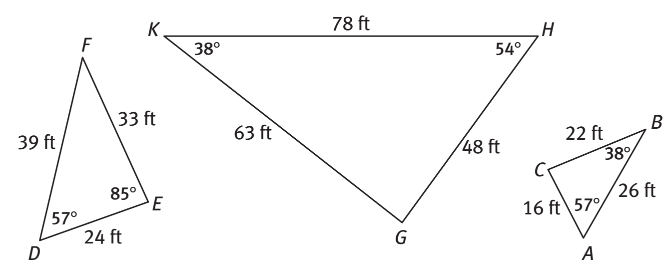
My Notes

LESSON 20-1 PRACTICE

10. Are the triangles below similar? If so, explain why and write a similarity statement. If not, explain why not.



Use the figure below for Items 11–13.



11. Identify the pair of similar triangles in the figure. Explain your answer.

12. Write a similarity statement for the triangles you identified in Item 11. Is there more than one correct way to write the statement?

13. What are the pairs of corresponding sides in the triangles you identified in Item 11?

14. **Construct viable arguments.** Malia is a jewelry designer. She created two silver triangles that she would like to use as earrings, but she is not sure if the two triangles are similar. One triangle has angles that measure 51° and 36°. The other triangle has angles that measure 36° and 95°. Is it possible to determine whether or not the triangles are similar? Justify your answer.

Lesson 20-2
Properties and Conditions of Similar Triangles

My Notes

Learning Targets:

- Determine whether triangles are similar given side lengths or angle measures.
- Calculate unknown side lengths in similar triangles.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Identify a Subtask, Visualization

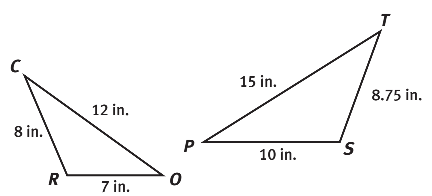
Two figures are similar if the corresponding angles are congruent and the corresponding sides are proportional. However, only one of these conditions is necessary in order to conclude that figures are congruent.

> **Necessary Condition for Similarity**
> When two triangles satisfy at least one of the following conditions, then they are similar.
> (1) The corresponding angles are congruent.
> (2) The corresponding sides are proportional.

The ratio of two corresponding sides of similar triangles is called the ***scale factor***.

1. What appears to be the scale factor for the similar triangles you created In Lesson 20-1 using the data collected when the mirror was 8 feet from the wall? Support your answer using corresponding sides for the similar triangles.
   The scale factor is $\frac{2}{3}$.

2. The triangles shown here are similar.



   a. Name the transformation that can help you identify the corresponding parts of the triangles.
   Rotation; for example, rotate $\triangle PST$ clockwise so that $\overline{ST}$ is horizontal.

   b. Write a similarity statement for the triangles.
   $\triangle CRO \sim \triangle PST$

   c. Determine the scale factor for the two similar triangles. Show your calculations.
   $\frac{4}{5}$

ACTIVITY 20 Continued

Lesson 20-2

PLAN

Pacing: 2 class periods

Chunking the Lesson

#1–2	#3
Example A	#4–6

Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity
Give students 3 or 4 minutes to sketch and label a figure of two similar triangles. Be sure students label all side lengths. Emphasize that the drawing does not have to be perfect, but should show correct proportional side lengths. Debrief by asking students how they know their triangles are similar.

1–2 Visualization, Create Representations, Sharing and Responding, Think-Pair-Share, Group Presentation These items can be used to assess student understanding of similarity statements and scale factors. Group presentations can be used to organize a class discussion concerning the way in which the triangles are named in Item 2b. Students should understand that there are actually six correct responses to this item.

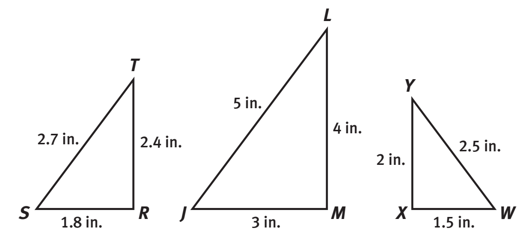
ELL Support

To support students' language acquisition, monitor their listening skills and understanding as they participate in group discussions. Carefully group students to ensure that all group members participate in and learn from collaboration and discussion.

3 Look for a Pattern, Visualization, Identify a Subtask, Group Presentation Only two of the three triangles in the given figure are similar. Students must realize that they should find similar triangles by comparing the ratios of corresponding side lengths. Again, group presentations will bring out several of the possible correct responses in Item 3b.

My Notes

3. Consider the three triangles below.



a. Compare ratios to identify any similar triangles.
$\frac{5}{2.5} = \frac{4}{2} = \frac{3}{1.5}$, so $\triangle JLM$ is similar to $\triangle WYX$.

b. Write a similarity statement to identify the similar triangles.
$\triangle JLM \sim \triangle WYX$

c. State the scale factor for the similar triangles.
The scale factor is 2.

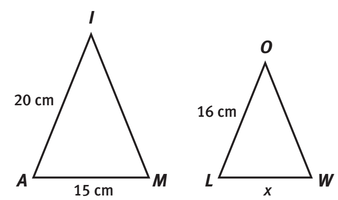
d. What are the pairs of corresponding angles of the similar triangles?
$\angle J$ corresponds to $\angle W$;
$\angle L$ corresponds to $\angle Y$;
$\angle M$ corresponds to $\angle X$.

Lesson 20-2
Properties and Conditions of Similar Triangles

The scale factor can be used to determine an unknown side length in similar figures.

Example A

Solve for x if $\triangle AIM \sim \triangle LOW$.



Step 1: Find the scale factor using known corresponding lengths.
The scale factor is $\frac{20\text{ cm}}{16\text{ cm}}$ or $\frac{5}{4}$.

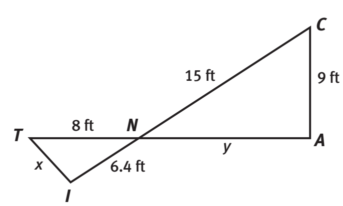
Step 2: Write a proportion using the scale factor.
$$\frac{5}{4} = \frac{15\text{ cm}}{x}$$

Step 3: Solve the proportion.
$$5x = 60.$$
$$x = 12$$

Solution: $x = 12$ cm

Try These A

Given $\triangle TIN \sim \triangle CAN$.



a. Determine the scale factor.
b. Solve for x and y.

My Notes

ACTIVITY 20 Continued

Example A Identify a Subtask, Create Representations, Marking the Text, Summarizing You may want to model the example on this page and employ a reading strategy such as Marking the Text or Summarizing to call attention to the main steps for finding an unknown side length in one of two similar triangles.

Teacher to Teacher

This example offers an ideal opportunity to discuss reasonableness of answers with students. To do so, ask them if the solution, $x = 12$ cm, seems reasonable and why. Invite students to share different ways of assessing the reasonableness of the answer. Some students may realize that side $\overline{LW}$ corresponds to side $\overline{AM}$ and, just as $\overline{AM}$ is a bit shorter than $\overline{AI}$, $\overline{LW}$ should be a bit shorter than $\overline{LO}$. Since 12 cm is a bit less than 16 cm, the answer seems reasonable.

Try These A

a. $\frac{8}{15}$
b. $x = 4.8$ ft; $y = 12$ ft

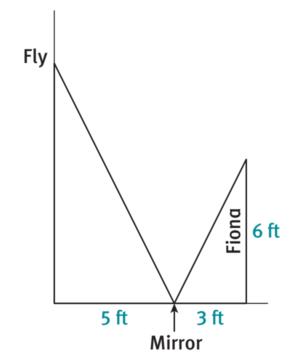
4–5 Visualization, Identify a Subtask, Create Representations, Think-Pair-Share These items provide students the opportunity to apply proportional reasoning to solve problems. Students must analyze the given information to label each diagram, set up a proportion, and solve for each length. The examples of measuring objects using similar triangles is a technique known as *indirect measurement*. It is often used to determine lengths that cannot easily be measured directly.

My Notes

Lesson 20-2
Properties and Conditions of Similar Triangles

4. Suppose that a fly has landed on the wall and a mirror is lying on the floor 5 feet from the base of the wall. Fiona, whose eye-level height is 6 feet, is standing 3 feet away from the mirror and 8 feet away from the wall. She can see the fly reflected in the mirror.

a. Use the information provided to label the distances on the diagram.

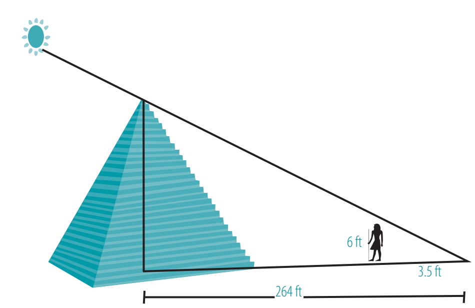


b. Show how to use the properties of similar triangles to calculate the distance from the floor to the observed fly.

10 ft

5. Model with mathematics. In his research, Thales determined that the height of the Great Pyramid could easily be calculated by the using the length of its shadow relative to the length of Thales's own shadow. Assume Thales was 6 feet tall and the shadow of the pyramid was 264 feet at the same time the shadow of Thales was 3.5 feet.

a. Using these data, label the distances on the diagram.



b. Determine the height of the Great Pyramid. Round your answer to the nearest tenth.

452.6 ft

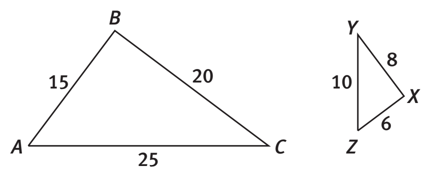
Lesson 20-2
Properties and Conditions of Similar Triangles

My Notes

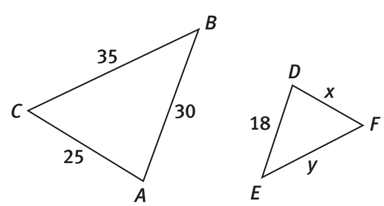
6. In $\triangle JKL$, $m\angle J = 32°$ and $m\angle K = 67°$. In $\triangle PQR$, $m\angle P = 32°$ and $m\angle Q = 67°$. Is $\triangle JKL \sim \triangle PQR$? Explain.

Check Your Understanding

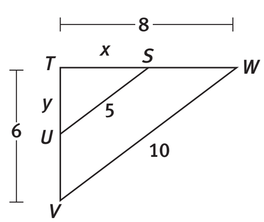
7. Are the two triangles shown below similar? If so, write a similarity statement and determine the scale factor. If not, explain why not.



8. Given $\Delta ABC \sim \Delta DEF$. Determine the value of x and y.



9. Given $\Delta TUS \sim \Delta TVW$. Determine the value of x and y.



ACTIVITY 20 Continued

6 Construct an Argument Students discover the AA similarity criterion for triangles through a numerical example. If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent and the triangles must be similar.

Check Your Understanding

Debrief students' answers to these items to assess whether students can work comfortably with similar triangles. Students should be able to identify similar triangles, write a similarity statement, and use proportionality to determine unknown side lengths. Debriefing students' solution methods will support struggling students who may need to see a variety of approaches in order to understand the material fully.

Answers

7. Yes; $\triangle ABC \sim \triangle ZXY$; 2.5
8. $x = 15$; $y = 21$
9. $x = 4$; $y = 3$

ASSESS

Students' answers to lesson practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 20-2 PRACTICE

10. $\triangle LOG \sim \triangle PIN$ and $\triangle ARM \sim \triangle BET$

11. $34\frac{5}{6}$ ft

12. 10 in.

13. The ratio of the perimeters equals the scale factor. For example, using the similar triangles in Item 11, the scale factor is 2.2, the perimeter of $\triangle ABC$ is 99 in., the perimeter of $\triangle RST$ is 45 in., and the ratio of the perimeters is $\frac{99}{45} = 2.2$.

14. Lucas is correct. Since the sum of the angle measures in a triangle is 180°, the third angles in the triangles must have the same measure. Since all three pairs of corresponding angles are congruent, the triangles are similar.

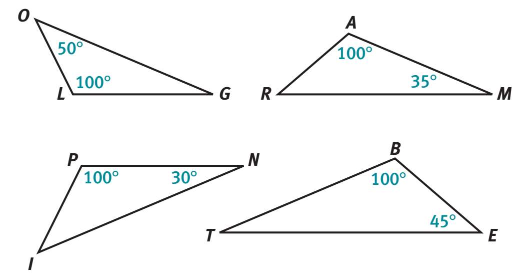
ADAPT

Review and debrief Lesson Practice items with students as an assessment of similarity concepts. Students who may need additional practice can be assigned problems from the Activity Practice.

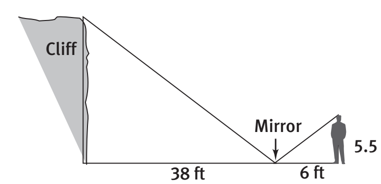
My Notes

LESSON 20-2 PRACTICE

10. Write similarity statements to show which triangles are similar.



11. Before rock climbing to the top of a cliff, Chen wants to know how high he will climb. He places a mirror on the ground and walks backward until he sees the top of the cliff in the mirror, as shown in the figure. What is the height of the cliff?



12. Given: $\triangle ABC \sim \triangle RST$
$AB = 44$ in., $BC = 33$ in., and $AC = 22$ in.
$RS = 20$ in. and $ST = 15$ in.
Find RT.

13. If two triangles are similar, how does the ratio of their perimeters compare to the scale factor? Use an example to justify your answer.

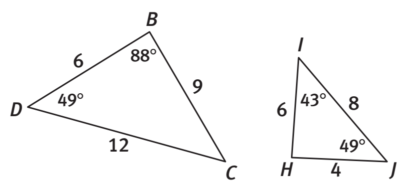
14. **Critique the reasoning of others.** Lucas claims, "If triangles have two pairs of congruent corresponding angles, then the third angles must also be congruent and the triangles must be similar." Is Lucas correct? Justify your answer.

ACTIVITY 20 PRACTICE

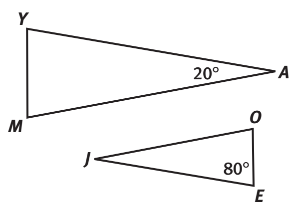
Write your answers on notebook paper.
Show your work.

Lesson 20-1

1. Determine whether the triangles are similar. If so, write a similarity statement. If not, explain why not.



2. If $\triangle JOE \sim \triangle AMY$, find the measure of each of the following angles.



a. $m\angle J$ **b.** $m\angle O$
c. $m\angle Y$ **d.** $m\angle M$

3. $\triangle ABC$ has side lengths 15 cm, 20 cm, and 25 cm. What could be the side lengths of a triangle similar to $\triangle ABC$?
   A. 7 m, 8 m, and 9 m
   B. 6 m, 8 m, and 10 m
   C. 5 cm, 10 cm, and 15 cm
   D. 30 mm, 40 mm, and 55 mm

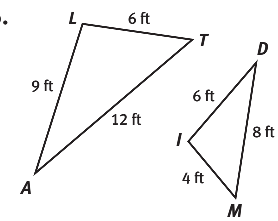
4. In $\triangle PQR$, $m\angle P = 27°$ and $m\angle R = 61°$. In $\triangle XYZ$, $m\angle Y = 92°$.
   a. Is it possible for $\triangle PQR$ to be similar to $\triangle XYZ$? Explain your reasoning.
   b. Can you conclude that $\triangle PQR$ is similar to $\triangle XYZ$? Why or why not?

5. Given that $\triangle ABC$ is similar to $\triangle GHJ$, which of the following statements must be true?
   A. Both triangles have the same side lengths.
   B. If $\triangle ABC$ has a right angle, then $\triangle GHJ$ has a right angle.
   C. The perimeter of $\triangle ABC$ is greater than the perimeter of $\triangle GHJ$.
   D. If $\triangle ABC$ has a side of length 2 cm, then $\triangle GHJ$ has a side of length 2 cm.

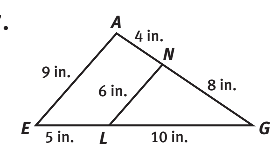
Lesson 20-2

For Items 6 and 7, determine whether the triangles shown are similar. If so, write a similarity statement for the triangles and determine the scale factor. If not, explain why not.

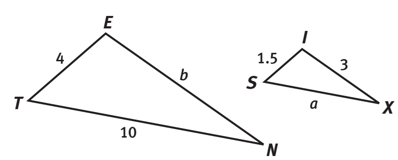
6.



7.



8. Given $\triangle SIX \sim \triangle TEN$, find a and b.



ACTIVITY 20 Continued

ACTIVITY PRACTICE

1. Yes; corresponding angles are congruent and corresponding sides are proportional. $\triangle BCD \sim \triangle HIJ$
2. **a.** 20°
   b. 80°
   c. 80°
   d. 80°
3. B
4. **a.** Yes; it is possible for corresponding angles to be congruent and for corresponding sides to be proportional.
   b. No; the two unknown angle measures in $\triangle XYZ$ may not be congruent to the two known angles of $\triangle PQR$, in which case the triangles would not be similar.
5. B
6. Yes; $\triangle ALT \sim \triangle DIM$; $\frac{3}{2}$
7. Yes; $\triangle GNL \sim \triangle GAE$; $\frac{2}{3}$
8. $a = 3.75$; $b = 8$

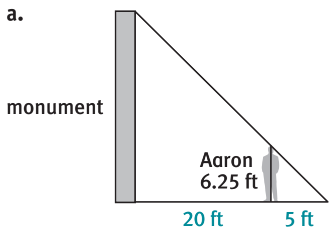
9. $p = 3$; $q = 2.6$

10. $x = 2$

11. **a.** 82°
b. 37°
c. 61°
d. 61°

12. $m\angle C = 34°$, $m\angle P = 90°$, $m\angle Q = 56°$, $m\angle R = 34°$

13. **a.**



b. 31.25 ft

14. 45

15. 180 meters

16. D

17. 36 cm

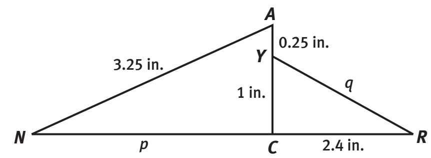
18. B

19. Since the sum of the measures of the angles of a triangle is 180°, each angle of an equiangular triangle must measure 60°. Given any two equiangular triangles, the corresponding angles are congruent (since they measure 60°) and therefore the triangles are similar.

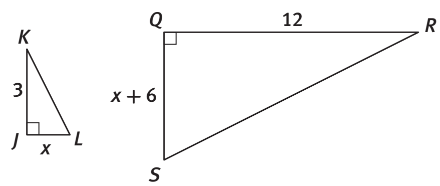
ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.

9. Given $\triangle CAN \sim \triangle CYR$, find p and q.



10. Given: $\triangle JKL \sim \triangle QRS$. Determine the value of x.



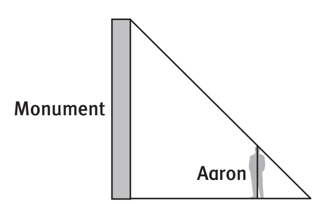
11. $\triangle MON \sim \triangle WED$, $m\angle M = 37°$, and $m\angle E = 82°$. Find the measure of each of the following angles.

a. $\angle O$ **b.** $\angle W$
c. $\angle N$ **d.** $\angle D$

12. Tell the measure of each angle of $\triangle ABC$ and $\triangle PQR$ if $\triangle ABC \sim \triangle PQR$, $m\angle A = 90°$, and $m\angle B = 56°$.

13. Aaron is 6.25 ft tall, and he casts a shadow that is 5 ft long. At the same time, a nearby monument casts a shadow that is 25 ft long.

a. Copy the figure and label the dimensions on the figure.



b. Determine the height of the monument.

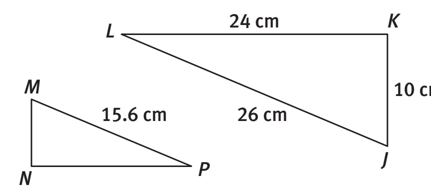
14. $\triangle ABC \sim \triangle DEF$ and the scale factor of $\triangle ABC$ to $\triangle DEF$ is $\frac{4}{3}$. If $AB = 60$, what is DE?

15. Sonia is 124 centimeters tall and casts a shadow that is 93 centimeters long. She is standing next to a tree that casts a shadow that is 135 meters long. How tall is the tree?

16. $\triangle STU \sim \triangle XYZ$, $ST = 6$, $SU = 8$, $XZ = 12$, and $YZ = 15$. What is the scale factor of $\triangle STU$ to $\triangle XYZ$?

A. $\frac{2}{5}$ **B.** $\frac{1}{2}$
C. $\frac{8}{15}$ **D.** $\frac{2}{3}$

17. In the figure, $\triangle JKL \sim \triangle MNP$. What is the perimeter of $\triangle MNP$?



18. $\triangle ABC \sim \triangle DEF$. $AB = 12$, $AC = 16$, $DE = 30$, and $DF = x + 5$. What is the value of x?

A. 30 **B.** 35
C. 40 **D.** 45

MATHEMATICAL PRACTICES
Look For and Make Use of Structure

19. An equiangular triangle is a triangle with three congruent angles. Explain why all equiangular triangles are similar.

Dilations

ACTIVITY 21

Alice's Adventures in Shrinking and Growing

Lesson 21-1 Stretching and Shrinking Geometric Figures

My Notes

Learning Targets:

- Investigate the effect of dilations on two-dimensional figures.
- Explore the relationship of dilated figures on the coordinate plane.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Predict and Confirm, Create Representations, Visualization

In the story *Alice's Adventures in Wonderland* written by Lewis Carroll, Alice spends a lot of time shrinking and growing in height. The height changes occur when she drinks a potion or eats a cake.

1. Complete the table by finding Alice's new height after she eats each bite of cake or drinks each potion.

Starting Height (inches)	Change in Height	New Height (inches)
56	$\frac{1}{8}$ times as tall	**7**
60	$\frac{2}{5}$ times as tall	**24**
60	1.5 times as tall	**90**
24	$\frac{5}{3}$ times as tall	**40**
30	2.2 times as tall	**66**

2. Each change in height resulted in a decrease or increase to Alice's starting height.

a. Alice's starting height decreased when it was multiplied by which two factors?

$\frac{1}{8}$ and $\frac{2}{5}$

CONNECT TO LITERATURE

Alice's Adventures in Wonderland is a novel published in 1865. It is the story of a young girl named Alice who wants to escape being bored by adulthood. In a dream she follows a white rabbit and falls down a deep tunnel and the adventure begins.

Common Core State Standards for Activity 21

8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

8.G.A.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

ACTIVITY 21

Guided

Activity Standards Focus

In earlier activities, students explored transformations that are rigid motions (translations, reflections, and rotations). In this activity, students expand their understanding of transformations to include a non-rigid motion. Specifically, students explore the effects of a dilation, and learn how to determine the scale factor of a dilation.

Lesson 21-1

PLAN

Materials

- ruler

Pacing: 2 class periods

Chunking the Lesson

#1–2 #3
#4 Example A
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity

Ask students to write examples of real-world situations in which the size of an object changes but its shape stays the same. If students have trouble getting started, suggest the example of making an enlargement on a photocopier. Have students share their ideas with the class, and then explain that this lesson focuses on a transformation that creates figures with the same shape but different sizes.

1–2 Look for Patterns, Predict and Confirm, Visualization, Construct an Argument, Group Presentation

Students complete the table by multiplying the starting height by the change in height. After completing the table, students are asked to determine the values that will make the new height smaller or larger than the original. Students should recognize that multiplying by a number greater than 1 will make the height larger, while multiplying by a positive number less than 1 will decrease the height. Students should not include 1 as a multiplier because 1 is the multiplicative identity and will not change the height.

Debrief students' work to be sure they use appropriate precision. In Item 2b, it is important that students understand the difference between all numbers less than 1 and *positive* numbers less than 1 (or numbers between 0 and 1).

Differentiating Instruction

To help with students' understanding in Item 2b, ask students what the result would be if the change in height were -2, which is a number less than 1. While the new height would be -120 inches, a height that is smaller than the starting height, students should realize that this is not an appropriate value for a height.

My Notes

b. Write a conjecture regarding the number you multiply by to decrease Alice's height.

Alice's height will decrease if you multiply her starting height by a number between 0 and 1.

c. Confirm your conjecture by providing two additional examples that show that Alice's starting height decreases.

Sample answer: $60 \text{ in.} \times \frac{1}{2} = 30 \text{ in.}$

$60 \text{ in.} \times \frac{1}{4} = 15 \text{ in.}$

d. Write a conjecture regarding the number you multiply by to increase Alice's starting height.

Alice's height will increase if you multiply the starting height by a number greater than 1.

e. Confirm your conjecture regarding Alice's increase in height by providing two additional examples that show that Alice's starting height increases.

Sample answer: $60 \text{ in.} \times 1.1 = 66 \text{ in.}$

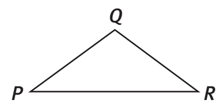
$60 \text{ in.} \times 1.2 = 72 \text{ in.}$

Alice's height changes—shrinking and growing—are a type of transformation known as a dilation.

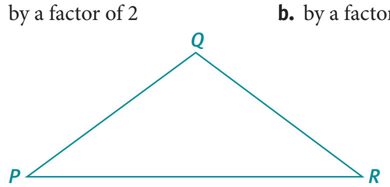
Lesson 21-1
Stretching and Shrinking Geometric Figures

A ***dilation*** is a transformation where the image is *similar* to the preimage; the size of the image changes but the shape stays the same.

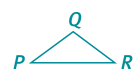
3. **Use appropriate tools strategically.** Given the preimage of $\triangle PQR$ below, use a ruler to draw the image of $\triangle PQR$ if it is dilated:



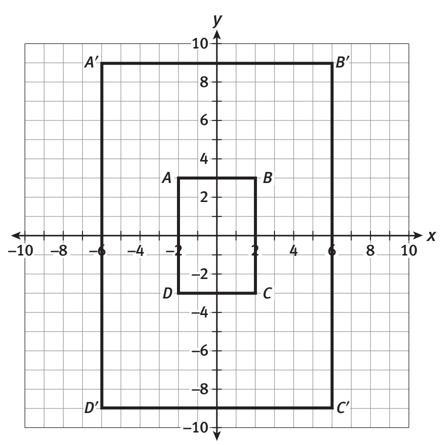
a. by a factor of 2



b. by a factor of $\frac{1}{2}$



4. Rectangles $ABCD$ and $A'B'C'D'$ are shown on the coordinate plane with the ***center of dilation*** at the origin, O.



My Notes

MATH TERMS

A **dilation** is a transformation that changes the size but not the shape of an object.

MATH TERMS

The **center of dilation** is a fixed point in the plane about which all points are expanded or reduced. It is the only point under a dilation that does not move.

3 Visualization, Create Representations, Use Manipulatives, Interactive Word Wall, Think-Pair-Share After discussing the meaning of *dilation*, the term should be added to the Interactive Word Wall. Take this opportunity to compare dilations to the rigid transformations that students investigated in previous activities. As students use a ruler to draw the dilations of the triangle in Item 3, it may be helpful to remind students that the image will be a triangle that is similar to the pre-image. This means the angles in $\triangle PQR$ will have the same measures as the angles in the dilation images.

TEACHER TO TEACHER

As a variation of Item 3, ask different members in each group to use different units when measuring the side lengths of the triangle. For example, Student 1 might measure each side of the triangle in centimeters, while Student 2 measures in inches and Student 3 measures in millimeters. After drawing the dilation image using the given factor, have students compare their image triangles to one another. These triangles should all be congruent.

4 Activating Prior Knowledge, Predict and Confirm, Look for a Pattern, Group Presentation Students use the grid to determine the lengths of each side of both rectangles. It may be beneficial to remind students that the opposite sides of rectangles are congruent. After students have completed Part a, you might ask students how they could verify that the two rectangles are similar.

Developing Math Language

This lesson introduces the term *dilation*. Prompt students to start using this term in their groups. Monitor group discussions to ensure that all members of the group are participating and that each member understands the language and terms used in the discussion.

Remind students that in *similar figures*, corresponding sides are proportional and corresponding angles are congruent.

Point out that the *center of dilation* is a point in a dilation that does not move. It is the point about which all other points are reduced and expanded.

Differentiating Instruction

Support If students are unsure about how to find the required side lengths, remind them that they can find the length of a horizontal or vertical line segment on the coordinate plane by simply counting the number of units from one endpoint of the segment to the other.

Extend Have students write symbolic notation for the dilation. Then ask them to verify that $(x, y) \rightarrow (3x, 3y)$ is the correct notation by checking that it works for specific points of $ABCD$ and $A'B'C'D'$.

My Notes

a. Determine the length of each side of rectangles $ABCD$ and $A'B'C'D'$.

Side	Length (in units)
$\overline{AB}$	4
$\overline{BC}$	6
$\overline{CD}$	4
$\overline{AD}$	6

Side	Length (in units)
$\overline{A'B'}$	12
$\overline{B'C'}$	18
$\overline{C'D'}$	12
$\overline{A'D'}$	18

b. Describe the relationship between the side lengths of rectangle $ABCD$ and rectangle $A'B'C'D'$.
Sample answer: The side lengths of rectangle *A′B′C′D′* are 3 times greater than the coordinates of rectangle *ABCD*.

c. Determine the coordinates of each of the vertices of both rectangles.

Rectangle *ABCD*		Rectangle *A′B′C′D′*	
A	(−2, 3)	A'	(−6, 9)
B	(2, 3)	B'	(6, 9)
C	(2, −3)	C'	(6, −9)
D	(−2, −3)	D'	(−6, −9)

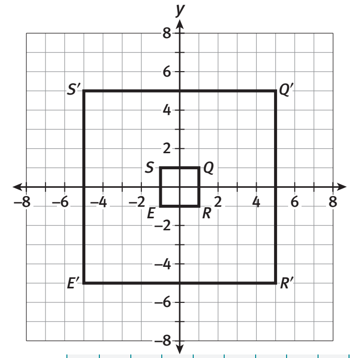
d. Describe the relationship between the coordinates of the vertices of $ABCD$ and the coordinates of the vertices of $A'B'C'D'$.
Sample answer: The coordinates of rectangle *A′B′C′D′* are 3 times greater than the coordinates of rectangle *ABCD*.

e. The point $\left(\frac{1}{3}, -3\right)$ is a point on rectangle $ABCD$. What are the coordinates of the image of the point on $A'B'C'D'$? Explain how you determined your answer.
(1, −9); I multiplied both the *x*- and *y*-coordinates by 3.

My Notes

Example A

Quadrilateral $SQRE$ is dilated to quadrilateral $S'Q'R'E'$ as shown on the coordinate plane. What is the relationship between the side lengths, perimeter, and area of the two figures?



Step 1: Compare the side lengths of corresponding sides of quadrilateral $S'Q'R'E'$ to quadrilateral $SQRE$.

$$\frac{S'Q'}{SQ} = \frac{10}{2} = \frac{5}{1}; \quad \frac{S'E'}{SE} = \frac{10}{2} = \frac{5}{1}$$
$$\frac{E'R'}{ER} = \frac{10}{2} = \frac{5}{1}; \quad \frac{R'Q'}{RQ} = \frac{10}{2} = \frac{5}{1}$$

The side lengths of quadrilateral $S'Q'R'E'$ are 5 times as great as the side lengths of quadrilateral $SQRE$.

Step 2: Find the perimeter of each quadrilateral. Then write the ratio of the perimeter of quadrilateral $S'Q'R'E'$ to the perimeter of quadrilateral $SQRE$.

Perimeter of quadrilateral $SQRE = 8$ units
Perimeter of quadrilateral $S'Q'R'E' = 40$ units

$$\text{ratio} : \frac{\text{Perimeter of } S'Q'R'E'}{\text{Perimeter of } SQRE} = \frac{40}{8} = \frac{5}{1}$$

Solution: The perimeter of quadrilateral $S'Q'R'E'$ is 5 times as great as that of quadrilateral $SQRE$.

Step 3: Find the area of each quadrilateral. Then write the ratio of the area of quadrilateral $S'Q'R'E'$ to the area of quadrilateral $SQRE$.

Area of quadrilateral $SQRE = 4$ square units
Area of quadrilateral $S'Q'R'E' = 100$ square units

$$\text{ratio} : \frac{\text{Area of } S'Q'R'E'}{\text{Area of } SQRE} = \frac{100}{4} = \frac{25}{1}$$

Solution: The area of quadrilateral $S'Q'R'E'$ is 25 times as great as that of quadrilateral $SQRE$.

READING MATH

The fraction bar in a ratio is read aloud as "to." For example, the ratio $\frac{4}{1}$ is read as "4 to 1."

As you discuss Example A, make notes about the notation and vocabulary used so you can review them later to aid your understanding of dilating geometric figures.

Example A Activating Prior Knowledge, Think-Pair-Share Students are asked to determine the ratio of the side lengths, perimeters, and areas of the quadrilaterals. Some students may choose to organize their answers for this type of problem in tabular form. In addition, when writing the ratios, students can write their responses using any of the following three formats:

(perimeter of $S'Q'R'E'$) : (perimeter of $SQRE$)

perimeter of $S'Q'R'E'$ to perimeter of $SQRE$

$$\frac{\text{perimeter of } S'Q'R'E'}{\text{perimeter of } SQRE}$$

Similar formats can be used for the ratios of the areas.

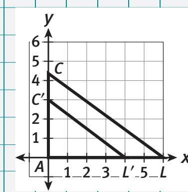
Check Your Understanding

Debrief students' answers to these items as a formative assessment to check whether students are able to connect dilations to their effect on the side lengths, perimeter, and area of a figure.

Answers

5. Since the perimeter ratio is comparing the image to the pre-image, and the ratio is a value greater than 1, then the image has a greater perimeter than the pre-image.
6. Corresponding sides must be in proportion.
7. Bradley; only one element, either radius or diameter, defines the dimensions of a circle. Susan is incorrect because the length and width of a rectangle defines its dimensions.

My Notes



MATH TIP

The area of a triangle can be found using the formula

Area $= \frac{1}{2}$ base $\times$ height.

In a right triangle, the legs can be used as the base and height.

Try These A

Triangle ALC is dilated to $\triangle AL'C'$ as shown on the coordinate plane. Triangle ALC has vertices $A(0, 0)$, $L(6, 0)$, $C(0, 4\frac{1}{2})$. The length of $\overline{C'L'}$ is 5 units.

a. Substitute known values into the proportion to find the length of $\overline{CL}$.

$$\frac{LA}{L'A} = \frac{CL}{C'L'}$$
$$\frac{6}{4} = \frac{CL}{5}$$
$$7.5 = CL$$

b. Determine the ratio of the perimeter of $\triangle AL'C'$ to the perimeter of $\triangle ALC$.

ratio: $\frac{\text{Perimeter of } \triangle AL'C'}{\text{Perimeter of } \triangle ALC} = \frac{12}{18} = \frac{2}{3}$

c. Determine the ratio of the area of $\triangle AL'C'$ to the area of $\triangle ALC$.

ratio: $\frac{\text{Area of } \triangle AL'C'}{\text{Area of } \triangle ALC} = \frac{6}{13.5} = \frac{4}{9}$

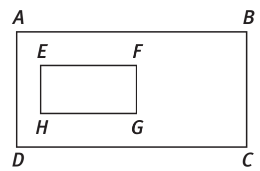
Check Your Understanding

5. Triangle ABC is dilated to $\triangle A'B'C'$. The ratio of the perimeter of $\triangle A'B'C'$ to the perimeter of $\triangle ABC$ is $\frac{4}{1}$. Explain how you can use this information to determine if the image has a larger or smaller perimeter than the preimage.
6. Square $TUVW$ is enlarged to form square $T'U'V'W'$. What must be true about the relationship between corresponding sides for the enlargement to be considered a dilation?
7. **Reason abstractly.** Bradley states that in theory circles with different diameters are all dilations of each other. Susan states that in theory rectangles with different side lengths are all dilations of each other. Do you agree with either, both, or neither statement? Explain your reasoning.

My Notes

LESSON 21-1 PRACTICE

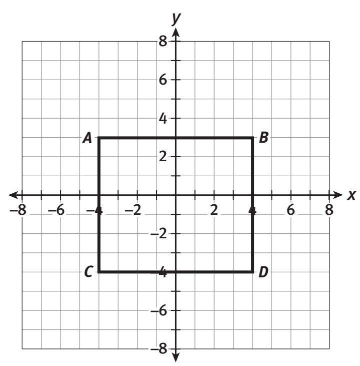
8. Rectangle *ABCD* is dilated to the rectangle *EFGH*. It is given that $AB = 48$ ft, $BC = 24$ ft, and $FG = 10$ ft.



a. Determine the ratio between corresponding side lengths.
b. Explain how knowing the ratio of corresponding side lengths helps you to determine the length of *EF*.
c. Find the length of *EF*.

9. A right triangle has vertices $A(0, 0)$, $B(10, 0)$, and $C(10, 24)$. The triangle is dilated so that the ratio between corresponding side lengths of the preimage to the image is $\frac{3}{1}$. Explain the effect on the area and perimeter of the dilated triangle.

10. Reason quantitatively. Figure *ABCD* is shown on the coordinate plane. Suppose a graphic designer wants to dilate the figure so that the resulting image has a smaller area than figure *ABCD*. Describe a way the designer can achieve this type of dilation.



11. Construct viable arguments. Alice's teacher explains that all circles are similar and asks the class to investigate relationships between a circle with radius 4 cm and a circle with radius 6 cm. Dante claims that the ratio of the areas of the circles is $\frac{4}{9}$, while Louisa claims that the ratio of the areas is 2.25 to 1. Who is correct? Give evidence to support the claim.

ACTIVITY 21 Continued

ASSESS

Students' answers to lesson practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 21-1 PRACTICE

8. a. $\frac{BC}{FG} = \frac{24}{10} = \frac{2.4}{1}$
b. Sample answer: The ratio tells me that *EF* is 2.4 times shorter in length than *AB*.
c. 20 ft

9. The area is 9 times as large and the perimeter is 3 times as large.

10. Multiply each side length by the same factor, a number between 0 and 1.

11. They are both correct. Evidence may vary. Sample: Dante is comparing the areas using a ratio of smaller circle to larger circle, while Louisa is comparing the areas using a ratio of larger circle to smaller circle. Area of the circle with radius 4 cm is $16\pi \approx 50.24$ and area of the circle with radius 6 cm is $36\pi \approx 113.04$. The ratio of the areas is $\frac{50.24}{113.04} = \frac{4}{9}$ or $\frac{113.04}{50.24} = 2.25$

ADAPT

Check students' work to ensure that they can determine the effect of a dilation on the side lengths, perimeter, and area of a figure. If students need additional practice, ask them to draw a rectangle on a coordinate plane. Then have them multiply the coordinates of each vertex by 2 and draw the resulting rectangle. Have students determine the ratio of the side lengths, perimeters, and areas of the two rectangles.

Lesson 21-2

PLAN

Pacing: 2 class periods

Chunking the Lesson

#1–2 Example A
Check Your Understanding
#6–7 #8–9
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity

Have students spend a few minutes doing a Quickwrite in which they summarize what they know about dilations. Have students share their work with the class to consolidate their understanding of dilations, activate prior knowledge, and review key terminology associated with dilations.

1–2 Look for Patterns, Create Representations, Think-Pair-Share, Group Presentation Help students make a connection to the scale factor of similar triangles that they learned in the previous activity. To determine the scale factor in Item 1, students may set up ratios using the corresponding sides of the triangles. It is important for students to recognize that a scale factor greater than 1 will result in an enlargement of the pre-image, while a scale factor less than 1 will result in a reduction. Students should also make the connection that the two different scale factors are reciprocals of each other.

Monitor students' group discussions to ensure that complex mathematical concepts are being verbalized precisely, using terms such as *enlargement* and *reduction*, and that all group members are actively participating in discussions through sharing ideas and through asking and answering questions appropriately.

Lesson 21-2
Effects of Scale Factor

My Notes

MATH TERMS

The **scale factor of dilation** is the factor by which each linear measure of the figure is multiplied.

Learning Targets:

- Determine the effect of the value of the scale factor on a dilation.
- Explore how scale factor affects two-dimensional figures on a coordinate plane.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Graphic Organizer, Create Representations

In the story *Alice's Adventures in Wonderland*, when Alice drinks a potion or eats a cake, she physically becomes taller or shorter, depending on a given factor. When this height change occurs, Alice changes size, but she does not change shape. Each dimension of her body is proportionally larger or smaller than her original self.

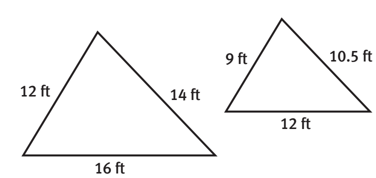
The factor by which Alice's height is changed, or dilated, is known as a **scale factor**.

The ***scale factor of dilation***, typically represented by the variable k, determines the size of the image of a dilated figure.

If $0 < k < 1$, then the image will be smaller than the original figure. In this case, the dilation is called a **reduction**.

If $k > 1$, then the image will be larger than the original figure, and dilation is called an **enlargement**.

1. Consider the similar triangles shown.



a. By what scale factor is the smaller triangle enlarged? Explain why the factor given must result in an enlargement.
Scale factor $= \frac{4}{3}$; This scale factor will result in an enlargement because $\frac{4}{3}$ is greater than 1.

b. By what scale factor is the larger triangle reduced? Explain why the factor given must result in a reduction.
Scale factor $= \frac{3}{4}$; This scale factor will result in a reduction because $\frac{3}{4}$ is less than 1.

c. What is the relationship between the two scale factors?
They are reciprocals of each other.

Lesson 21-2
Effects of Scale Factor

My Notes

2. Suppose a point with coordinates (x, y) is a vertex of a geometric figure and that figure is dilated by a scale factor of k with the ***center of dilation*** at the origin.

a. Create an ordered pair to represent the coordinates of the corresponding point on the image.
(kx, ky)

b. Predict the size of the image as it compares to the preimage if k is 10.
Sample answer: The image would be an enlargement of the preimage, or 10 times greater in size.

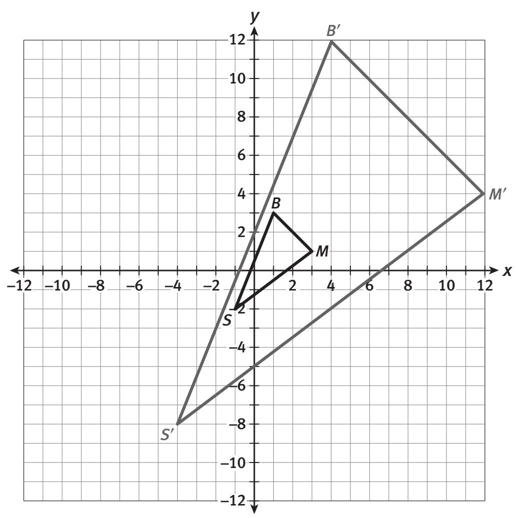
c. Predict the size of the image as it compares to the preimage if k is 0.5.
Sample answer: The image would be a reduction of the preimage, or one-half times smaller in size.

MATH TERMS

The **center of dilation** is a fixed point in the plane about which all points are expanded or reduced. It is the only point under a dilation that does not move. The center of dilation determines the location of the image.

Example A

Triangle $S'B'M'$ is a dilation of $\triangle SBM$ with a scale factor of 4. Using the coordinates of the vertices of $\triangle SBM$, determine the coordinates of the vertices of $\triangle S'B'M'$. Then plot $\triangle S'B'M'$ on the coordinate plane.



Step 1: Determine if the dilation is a reduction or enlargement.
Since the scale factor is 4 and $4 > 1$, the dilation is an enlargement.

Step 2: Multiply the coordinates of the vertices of $\triangle SBM$ by the scale factor.
$\triangle SBM$: $S(-1, -2)$, $B(1, 3)$, $M(3, 1)$
Multiply each coordinate by 4.
$\triangle S'B'M'$: $S'(-4, -8)$, $B'(4, 12)$, $M'(12, 4)$

Step 3: Plot the coordinates of the vertices of $\triangle S'B'M'$ on the coordinate plane.

Example A Visualization, Create Representations, Think-Pair-Share, Graphic Organizer This example will help students connect several essential concepts: dilations, scale factors, ratios, enlargements and reductions, similarity, and changes to coordinates under a dilation. As you discuss the example, be sure students understand that a scale factor greater than 1 should result in an image of $\triangle SBM$ that is larger than $\triangle SBM$. Also, it is important for students to realize that $\triangle SBM$ is similar to $\triangle S'B'M'$.

Check Your Understanding

Debrief students' responses to these items to ensure that they understand the connection between the ratio of the side lengths of dilated figures and the scale factor of the dilation. You can also use these items to check that students recognize when a dilation is an enlargement or a reduction. Discussing these items before continuing with the lesson will benefit students who may still be working to master these concepts.

Answers

3. The ratio of side lengths of dilated figures is equal to the scale factor of dilation.
4. Reduction, because the scale factor, 0.12, is less than 1.
5. There will not be any changes. The point will remain (0, 0).

Try These A

a. Suppose the scale factor of dilation of $\triangle SBM$ in Example A is $\frac{1}{2}$. Determine if the resulting image, $\triangle S'B'M'$, will be a reduction or an enlargement of $\triangle SBM$. Then, determine the coordinates of $\triangle S'B'M'$.
Since the scale factor is $\frac{1}{2}$, and $\frac{1}{2} < 1$, then the dilation is a reduction.
$\triangle S'B'M'$: $S'\left(-\frac{1}{2}, -1\right)$, $B'\left(\frac{1}{2}, \frac{3}{2}\right)$, $M'\left(\frac{3}{2}, \frac{1}{2}\right)$

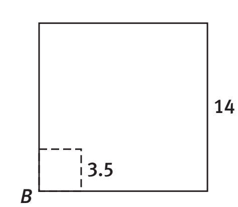
b. Figure $A'B'C'D'$ is a dilation of figure $ABCD$ with a scale factor of 5. Given the coordinates of the vertices of $A(0, 0)$, $B(0, 2)$, $C(-2, -2)$, $D(-2, 0)$, determine the coordinates of the vertices of figure $A'B'C'D'$.
$A'(0, 0)$, $B'(0, 10)$, $C'(-10, -10)$, $D'(-10, 0)$

Check Your Understanding

3. Compare the ratio of the side lengths of figure $A'B'C'D'$ and figure $ABCD$ to the scale factor in Try These part b. Make a conjecture about the ratio of side lengths of dilated figures and the scale factor of dilation.
4. Triangle $P'Q'R'$ is a dilation image of $\triangle PQR$. The scale factor for the dilation is 0.12. Is the dilation an enlargement or a reduction? Explain.
5. **Make use of structure.** A geometric figure contains the point (0, 0) and is dilated by a factor of m with the center at the origin. What changes will occur to the point (0, 0)?

The scale factor of dilation describes the size change from the original figure to the image. The scale factor can be determined by comparing the ratio of corresponding side lengths.

6. The solid line figure shown is a dilation of the figure formed by the dashed lines. Describe a method for determining the scale factor used to dilate the figure.



Compare corresponding side lengths: $\frac{14}{3.5} = \frac{4}{1}$. The scale factor is 4.

Lesson 21-2
Effects of Scale Factor

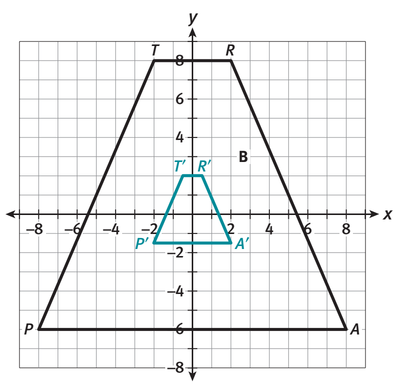
My Notes

7. **Critique the reasoning of others.** Josie found the scale factor in Item 6 to be $\frac{1}{4}$. Explain why Josie got the wrong scale factor.
   She did not recognize that the dilation was an enlargement. She compared the side length of the smaller figure to that of the larger figure.

There exists a relationship between the area of dilated figures and the perimeter of dilated figures.

8. Make a prediction about the effect of the scale of dilation on the area and perimeter of two figures.
   Sample answer: The perimeter changes by the same factor as the scale factor, and the area changes by the square of the scale factor.

9. Trapezoid *TRAP*, shown on the coordinate plane, has vertices $(-2, 8)$, $(2, 8)$, $(8, -6)$, $(-8, -6)$. Suppose trapezoid TRAP is dilated by a scale factor of $\frac{1}{4}$.



   a. Plot and label the vertices of the image $T'R'A'P'$.
   $T'\left(-\frac{1}{2}, 2\right)$, $R'\left(\frac{1}{2}, 2\right)$, $A'(2, -1.5)$, $P'(-2, -1.5)$

   b. Determine the area of trapezoids *TRAP* and $T'R'A'P'$.
   area of *TRAP* = 140 square units
   area of $T'R'A'P'$ = 8.75 square units

   c. What is the ratio of the area of *TRAP* to the area of $T'R'A'P'$?
   $\frac{140}{8.75} = \frac{16}{1}$

   d. **Reason quantitatively.** Make a conjecture about the relationship between scale factor of dilation and the area of dilated figures.
   The area of the dilated figure changes by the square of the scale factor.

MATH TIP
The area of a trapezoid can be found using the formula Area $= \frac{1}{2}h(b_1 + b_2)$, where h is the height and b_1 and b_2 are the bases.

6–7 Visualization, Construct an Argument, Think-Pair-Share, Debriefing Have students share their methods for finding the scale factor in Item 6. It is important for students to understand that whenever they calculate a scale factor, they should set up the ratio so that it compares the new figure to the original figure. Equivalently, the ratio should compare the image to the pre-image. This will ensure that students are finding the correct scale factor and not its reciprocal.

8–9 Visualization, Predict and Confirm, Construct an Argument, Think-Pair-Share, Debriefing Students should use the grid to determine the height and length of each base. Students who do not yet know the Pythagorean Theorem or how to determine the distance between the points *T* and *P* or between *A* and *R*, cannot determine the perimeter of these trapezoids. For that reason, this item only asks students to determine the ratio of the areas.

Teacher to Teacher
As a conclusion to students' work with transformations, you may want to ask students to compare and contrast the four types of transformations they have seen: translations, reflections, rotations, and dilations.

Check Your Understanding

Debrief students' answers to these items as a formative assessment of the key ideas in this lesson. Check that students can determine the ratio of the perimeters and the ratio of the areas of dilated figures just by knowing the scale factor. This is a powerful idea for students, so be sure they understand that it is possible to know the ratio of the perimeters or areas of two figures without knowing the specific dimensions of the figures.

Answers

10. ratio of the perimeters $= k$; ratio of the areas $= k^2$

11. **a.** $\frac{2}{5}$, **b.** $\frac{4}{25}$

12. The pre-image and the image will be congruent. Examples will vary.

ASSESS

Students' answers to lesson practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 21-2 PRACTICE

13. **a.** $\frac{36}{24} = \frac{3}{2} = 1.5$

b. $\frac{9}{4}$

14. scale factor $= 3$

15. **a.** Check students' drawings. Vertices of the trapezoid are at $A'(-3, -3)$, $B'(-3, 3)$, $C'(6, 6)$, and $D'(6, -3)$

b. 3:1

c. 9:1

16. Scale factor $= \frac{7}{3}$ This is an enlargement because $\frac{7}{3} > 1$.

ADAPT

Students should be able to work comfortably with dilations, scale factors, similarity, and ratios of side lengths, perimeters, and areas. If students need additional work with these concepts, ask them to draw two rectangles on a coordinate plane that are related by a scale factor of 3. Then have students find the ratio of the rectangles' side lengths, perimeters, and areas.

My Notes

Check Your Understanding

10. Suppose a polygon is dilated by a scale factor of k. Write an expression for the ratio of the perimeters. Then, write an expression to represent the ratio of the areas.

11. A triangle is dilated by a scale factor of $\frac{2}{5}$.

a. What is the ratio of the perimeters?

b. What is the ratio of the areas?

12. **Construct viable arguments.** Suppose that a dilation is executed with a scale factor of 1. How would the preimage relate to the image? Using an example, justify your answer.

LESSON 21-2 PRACTICE

13. A rectangle has a perimeter of 24 ft. Following a dilation, the new perimeter of the rectangle is 36 ft.

a. Determine the scale factor of dilation.

b. What is the ratio of the areas?

14. A triangle has an area of 40 cm^2. Following a dilation, the new area of the triangle is 360 cm^2. What is the scale factor of dilation?

15. The vertices of trapezoid $ABCD$ are $A(-1, -1)$, $B(-1, 1)$, $C(2, 2)$, and $D(2, -1)$.

a. Draw the trapezoid and its dilation image for a dilation with center (0, 0) and scale factor 3.

b. Determine the ratio of the perimeter.

c. Determine the ratio of the areas.

16. **Make sense of problems.** Eye doctors dilate patients' pupils to get a better view inside the eye. If a patient's pupil had a 3.6-mm diameter before dilation and 8.4-mm diameter after dilation, determine the scale factor used to dilate the pupil. Explain why this created an enlargement.

MATH TIP

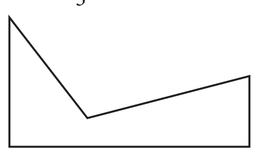
The area of a circle can be found using the formula Area $= \pi r^2$, where r is the radius of the circle.

ACTIVITY 21 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 21-1

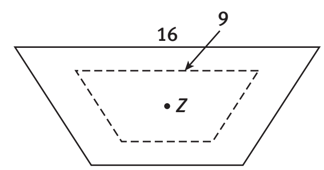
1. **Use appropriate tools strategically.** Sketch the dilation of the image of the figure below using a scale factor of $\frac{2}{3}$.



2. Does the size of a preimage increase or decrease when
   a. dilated by a factor greater than 1?
   b. dilated by a factor between 0 and 1?

3. The ratio of the area of $\triangle X'Y'Z'$ to the area of $\triangle XYZ$ is $\frac{2}{9}$. Explain how you can use this information to determine if the image is greater or smaller in area than the preimage.

4. The solid line figure is a dilation of the dashed line figure. Tell whether the dilation is an enlargement or a reduction. Then find the scale factor of the dilation.



5. Explain how dilations are different from other types of transformations you have studied.

6. If the radius of a circle is 24 ft, how many circles can be the dilations of this circle? Why?

Lesson 21-2

7. A dilation has a center (0, 0) and scale factor 1.5. What is the image of the point (−3, 2)?

8. A triangle has vertices (−1, 1), (6, −2), and (3, 5). If the triangle is dilated with a scale factor of 3, which of the following are the vertices of the image?
   A. (−3, 3), (18, −6), (9, 15)
   B. (3, 3), (18, 6), (9, 15)
   C. (−3, 3), (18, 6), (9, 15)
   D. (3, 3), (18, −6), (9, 15)

ACTIVITY 21 Continued

ACTIVITY PRACTICE

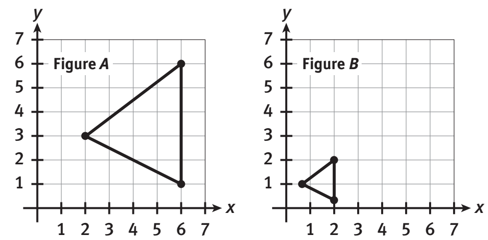
1. Check students' drawings.
2. **a.** increase **b.** decrease
3. Since the area ratio is comparing the image to the pre-image, and the ratio is a value less than 1, then the image has a smaller area than the pre-image.
4. Enlargement; scale factor $= \frac{16}{9}$
5. Sample answer: In a dilation (other than 1), the image has a different size.
6. An infinite number; since the circles are curved and only one element, either radius or diameter, defines their properties
7. (−4.5, 3)
8. A

9. C
10. A
11. 160 ft
12. $\frac{7}{13}$

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.

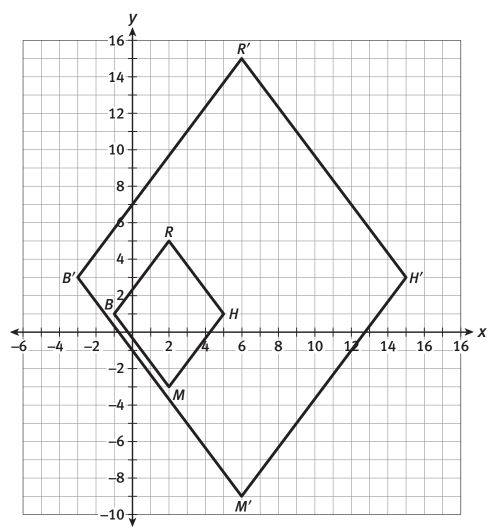
9. Figure *B* is the result of a dilation of Figure *A*.



What is the scale factor of dilation?

A. 3 **B.** 2
C. $\frac{1}{3}$ **D.** $\frac{1}{2}$

10. Rhombus *RHMB* has vertices (2, 5), (5, 1), (2, −3), and (−1, 1). This figure has been dilated to rhombus *R′H′M′B′*, as shown on the coordinate plane.



The area of rhombus *RHMB* is 24 square units. Which of the following is the area of rhombus *R′H′M′B′*?

A. 216 square units
B. 72 square units
C. 8 square units
D. 2.7 square units

11. The diagonals of rhombus *ABCD* are 6 ft and 8 ft. Rhombus *ABCD* is dilated to rhombus *RSTU* with the scale factor 8. What is the perimeter of rhombus *RSTU*?

MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

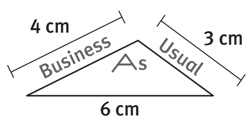
12. The endpoints of $\overline{AB}$ are *A*(78, 52) and *B*(26, −52). $\overline{AB}$ is dilated to $\overline{GH}$ with endpoints at *G*(30, 20) and *H*(10, −20). Then, $\overline{GH}$ is dilated to $\overline{PQ}$ with endpoints at *P*(42, 28) and *Q*(14, −28). If $\overline{AB}$ is dilated directly to $\overline{PQ}$, what will be the scale factor?

Similarity and Dilations
BUSINESS AS USUAL

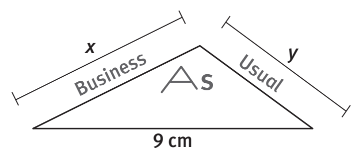
Embedded Assessment 3
Use after Activity 21

Liz is a commercial artist working for Business as Usual. The company specializes in small-business public relations. Liz creates appealing logos for client companies. In fact, she helped create the logo for her company. Business As Usual will use its logo in different sizes, with each design including a triangle similar to the one shown.

1. The advertisement and stationery letterhead–size logos are shown below with the measurements of some of the side lengths. Determine the missing measures of the sides.



Letterhead



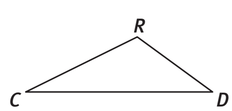
Advertisement

2. To create the triangles in the design, Liz wants to determine the measure of each angle in the designs. The advertisement logo is shown below including the measures of two of its angles. The business card logo will be similar to the advertisement so that $\triangle BAU \sim \triangle CRD$. Determine the measure of each angle.

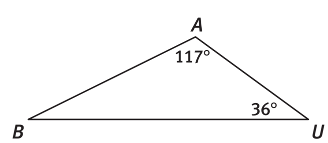
a. $m\angle C =$ _______

b. $m\angle R =$ _______

c. $m\angle D =$ _______



Business Card



Advertisement

Embedded Assessment 3

Assessment Focus

- Identify similar figures and find unknown measures
- Perform dilations on the coordinate plane
- Find perimeters and areas of similar figures

Materials

- calculator

Answer Key

1. $x = 6$ cm; $y = 4.5$ cm

2. **a.** $m\angle C = 27°$

b. $m\angle R = 117°$

c. $m\angle D = 36°$

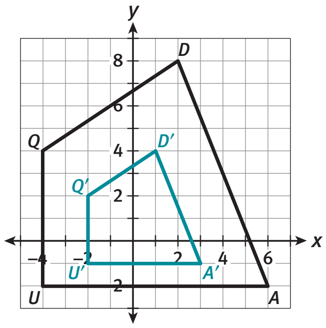
Common Core State Standards for Embedded Assessment 3

8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

8.G.A.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

8.G.A.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

3. a. $Q'(-2, 2), U'(-2, -1), A'(3, -1), D'(1, 4)$



b. $\frac{1}{2}$

c. $\frac{1}{4}$

4. The triangles are not similar. Explanations may vary. All three ordered pairs of $\triangle DEF$ are not dilated by the same value to get the corresponding ordered pairs of $\triangle ABC$.

Embedded Assessment 3

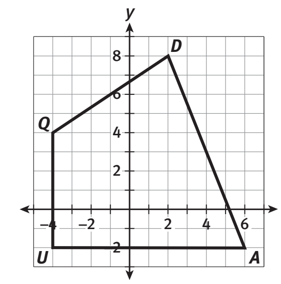
Use after Activity 21

Similarity and Dilations

BUSINESS AS USUAL

Liz tries to incorporate triangles and quadrilaterals into many of the logos she designs for her clients. She begins her layout by laying it out on a coordinate plane.

3. Quadrilateral *QUAD* is shown.

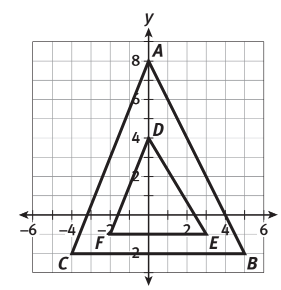


a. Quadrilateral $Q'U'A'D'$ is a dilation of *QUAD* with scale factor $\frac{1}{2}$. List the coordinates of $Q'U'A'D'$ and sketch the graph on a coordinate plane.

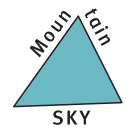
b. Determine the ratio of the perimeter of $Q'U'A'D'$ to the perimeter of *QUAD*.

c. Determine the ratio of the area of $Q'U'A'D'$ to the area of *QUAD*.

4. The coordinates of $\triangle ABC$ are $A(0, 8)$, $B(5, -2)$, and $C(-4, -2)$, and the coordinates of $\triangle DEF$ are $D(0, 4)$, $E(3, -1)$, and $F(-2, -1)$. Determine whether or not $\triangle ABC$ is similar to $\triangle DEF$. Defend your answer.



5. You have been chosen to work with Liz on a logo for a new client, Mountain Sky, a company that provides camping equipment and guides. Using either the logo design shown or a design of your own, recreate the design in sizes appropriate for a business card, business stationery letterhead, and an advertisement. Use properties of similar triangles to explain to Liz how you know the designs are dilations of the original. Include scale factors for each design.



5. Answers will vary depending on the sizes chosen by students for each of the three designs. All triangles should have the same angle measures.

Teacher to Teacher

You may wish to read through the scoring guide with students and discuss the differences in the expectations at each level. Check that students understand the terms used.

Unpacking Embedded Assessment 4

Once students have completed this Embedded Assessment, turn to Embedded Assessment 4 and unpack it with students. Use a graphic organizer to help students understand the concepts they will need to know to be successful on Embedded Assessment 4.

Embedded Assessment 3

Use after Activity 21

Similarity and Dilations

BUSINESS AS USUAL

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2a-c, 3a-c, 4, 5)	• Accurately finding side lengths and angle measures in similar triangles. • Accurately using dilations and scale factors.	• Finding side lengths and angle measures in similar triangles. • Using dilations and scale factors with few errors.	• Difficulty finding side lengths and angle measures in similar triangles. • Difficulty using dilations and scale factors.	• Little or no understanding of finding side lengths in similar triangles. • Little or no understanding of dilations.
Problem Solving (Items 3b-c, 4, 5)	• An appropriate and efficient strategy that results in a correct answer.	• A strategy that may include unnecessary steps but is correct.	• A strategy that results in some incorrect answers.	• No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 3a, 5)	• Modeling dilations accurately and clearly.	• Drawing similar figures correctly.	• Difficulty drawing similar figures accurately.	• Incorrectly transforming pre-images and drawing the images.
Reasoning and Communication (Items 4, 5)	• Using precise language to justify that two triangles are similar.	• Explaining why two triangles are similar.	• A confusing explanation of triangle similarity.	• An inaccurate explanation of triangle similarity.

The Pythagorean Theorem

ACTIVITY 22

Stop the Presses

Lesson 22-1 Pythagorean Theorem: Squares of Lengths

My Notes

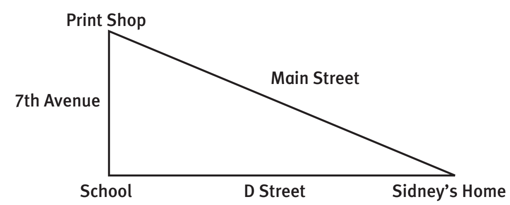
Learning Targets:

- Investigate the Pythagorean Theorem.
- Understand and apply the Pythagorean Theorem.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Marking the Text, Look for a Pattern, Interactive Word Wall, Create Representations

Jayla and Sidney are co-editors-in-chief of the school yearbook. They have just finished the final layouts of this year's edition. It is due at the print shop before it closes at 4 o'clock. The print shop is on her way home, so Jayla agrees to drop off the layouts at the print shop on the corner of 7th Avenue and Main Street. Sidney has a copy of the layouts with him to check one more time.

Jayla and Sidney part company at the front door of their school, which is located on the corner of 7th Avenue and D Street. Jayla walks toward the print shop on 7th Avenue and Sidney bikes toward his home on D Street.



When Jayla gets to the print shop, she notices that the set of layouts is missing the last three pages. She calls Sidney at home to see whether he can quickly bring his copy of the layouts to the print shop.

Sidney leaves his house at 3:45 P.M. and starts biking along Main Street to the print shop. As he is pedaling, he wonders how far it is to the print shop. His house is 12 blocks away from the school and the print shop is five blocks away from the school. He can travel, at the most, one block per minute on his bike.

1. Read the scenario carefully and discuss with a partner or in your group the key information provided and how you might use it. Then predict whether Sidney makes it to the print shop before it closes.
He will make it because it would take 17 minutes if he went on D Street and 7th Avenue, and Main Street looks a lot shorter.

The lengths of the three sides of any right triangle have a relationship that you could use to answer Item 1. It is one of the most useful properties you will use as you study mathematics.

ACTIVITY 22

Guided

Activity Standards Focus

In this activity, students investigate one of the most important theorems in mathematics, the Pythagorean Theorem. Students explore a proof of the theorem and then use the theorem to find unknown side lengths in right triangles. Note that additional applications of the theorem are covered in Activity 23, and students explore the converse of the theorem in Activity 24.

Lesson 22-1

PLAN

Materials

- graph paper
- scissors

Pacing: 1–2 class periods

Chunking the Lesson

#1 #2 #3–7
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity

Have students draw a right triangle. Then ask them to do a Quickwrite about the angles of the triangle. Ask students to share their work to bring out the idea that every right triangle has one right angle and two acute angles. Explain to the class that this activity focuses on an important relationship among the sides of a right triangle.

1 Marking the Text, Predict and Confirm, Shared Reading Most students will have limited knowledge of the Pythagorean Theorem at this point. Item 1 is the Predict part of a Predict and Confirm that will be completed later. There is no need to spend a lot of time on this item.

Common Core State Standards for Activity 22

8.G.B.6	Explain a proof of the Pythagorean Theorem and its converse.
8.G.B.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
8.G.B.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

2 Look for a Pattern, Interactive Word Wall, Create Representations, Use Manipulatives, Debriefing Be sure to debrief students after they do Part a of this question. Students should find that the two large squares in the two figures have the same area. This will be true for every case in the table. Have students present the examples they created for Case 4.

Developing Math Language

This lesson introduces the terms *leg* and *hypotenuse,* which should be added to the Interactive Word Wall. Add words to your classroom Word Wall regularly. Include math terms, academic vocabulary, and other words that students use regularly in their group or class discussions. To remind students to refer to the Word Wall, ask them to choose words to add. Another way to reinforce language acquisition is to have each student choose a word from the Word Wall and then pair-share for a few minutes to discuss its meaning and use.

My Notes

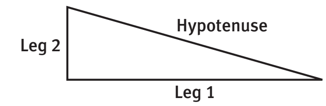
MATH TIP

It does not matter which leg is labeled Leg 1 and which is labeled Leg 2.

MATH TERMS

The **hypotenuse** is the longest side of a right triangle. It is the side that is opposite the right angle.

The **legs** of a right triangle are the two sides that create the right angle.



2. The ***hypotenuse*** of a right triangle is the side that is opposite the right angle. It is always the longest side of the triangle. The ***legs*** of a right triangle are the sides that form the right angle. Both Figures 1 and 2 have been formed using four congruent right triangles like the one above.

a. Use grid paper to cut out four congruent right triangles with Leg 1 equal to seven units and Leg 2 equal to two units. Recreate Figures 1 and 2 on another piece of graph paper by tracing your four congruent triangles and adding line segments to complete L and M. Then complete Case 1 in Table A at the bottom of this page.

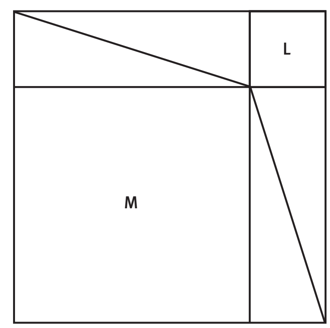


Figure 1

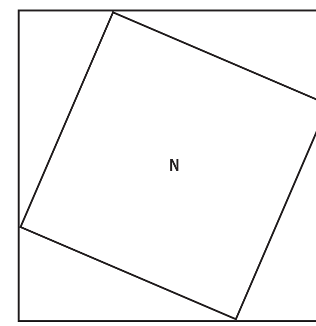


Figure 2

Table A

Case	Length Leg 1	Length Leg 2	Width Figure 1	Length Figure 1	Area Figure 1	Width Figure 2	Length Figure 2	Area Figure 2
1	7	2	9	9	81	9	9	81
2	6	3	9	9	81	9	9	81
3	4	3	7	7	49	7	7	49
4	Will vary	Will vary	Will vary	Will vary	Will vary	Will vary	Will vary	Will vary

b. Complete Cases 2 and 3 in Table A by cutting out triangles to recreate Figures 1 and 2 using the lengths given in the table.
[Answers in table]

c. Complete Case 4 in Table A by choosing your own leg lengths for a right triangle.
[Answers in table]

d. What do you notice about Figure 1 and Figure 2 in each case?
Figure 1 and Figure 2 have the same area because they are both 9 units by 9 units.

Lesson 22-1
Pythagorean Theorem: Squares of Lengths

My Notes

3. Now use the figures you drew for Cases 1 through 4 to complete the first seven columns (Case through Area of Shape M) in Table B. For Case 5, use the variables a and b as the lengths of Leg 1 and Leg 2.

Table B

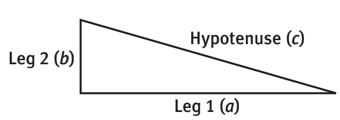
Case	Length Leg 1	Length Leg 2	Dimensions Shape L	Area Shape L	Dimensions Shape M	Area Shape M	Area Shape N
1	7	2	**2×2**	**4**	**7×7**	**49**	**53**
2	6	3	**3×3**	**9**	**6×6**	**36**	**45**
3	4	3	**3×3**	**9**	**4×4**	**16**	**25**
4	Will vary	Will vary	Will vary	Will vary	Will vary	Will vary	Will vary
5	a	b	**$b \times b$**	**b^2**	**$a \times a$**	**a^2**	**$a^2 + b^2$**

4. Describe the relationship between the areas of shapes L, M, and N and complete the Area of Shape N column of Table B.
The area of shape N is equal to the sum of the areas of shapes L and M.

5. Describe the lengths of the sides of shapes L, M, and N in terms of the sides of the right triangles.
The length of the side of shape L is the same as the length of Leg 2. The length of the side of shape M is the same as the length of Leg 1. The length of the side of shape N is the same as the length of the hypotenuse of the triangle.

6. Find the area of shapes L, M, and N in terms of the lengths of the sides of the right triangles.
The area of shape L = (the length of Leg 2)^2; the area of shape M = (the length of Leg 1)^2; and the area of shape N = (the length of the hypotenuse)^2.

7. Make use of structure. Use a for the length of Leg 1, b for the length of Leg 2, and c for the length of the hypotenuse to write an equation that relates the areas of shapes L, M, and N.
$a^2 + b^2 = c^2$



The relationship that you have just explored is called the ***Pythagorean Theorem***.

MATH TERMS

The **Pythagorean Theorem** states that the sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse.

CONNECT TO HISTORY

Although the Pythagorean Theorem is named for Pythagoras, a Greek mathematician who lived about 500 B.C.E., the ancient Babylonians, Chinese, and Egyptians understood and used this relationship even earlier.

3–7 Look for a Pattern, Create Representations, Think-Pair-Share Debriefing In these items, students work toward writing an equation that describes the relationship given by the Pythagorean Theorem. The items move students from the concrete to the abstract. For Item 4, make sure that students can write an equation that describes the relationship between the areas of the three shapes. Ensuring that students are successful with this step of the process will help them arrive at the key relationship, $a^2 + b^2 = c^2$, in Item 7.

Developing Math Language

Help students understand the definition of *Pythagorean Theorem.* As they examine the relationship in symbols, $a^2 + b^2 = c^2$, have them verbalize the relationship as the sum of the squares of the lengths of the legs are equivalent to the square of the length of the hypotenuse.

TEACHER TO TEACHER

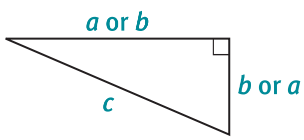
The Pythagorean Theorem makes an appearance in the 1939 film The *Wizard of Oz*. When the Scarecrow is given a brain, he shows off his new-found intelligence by reciting the theorem. However, he states the theorem incorrectly. Clips from this portion of the film are available online and you may wish to show the clip to students so they can spot the error.

Check Your Understanding

At this point in the activity, students should be comfortable writing the statement of the Pythagorean Theorem in a variety of ways and in a variety of situations. Debrief students' answers to these items as a formative assessment of these skills before students move on to applications of the theorem in the next lesson.

Answers

8.



9. 100 square units

10. $6^2 + 8^2 = 10^2$ or $36 + 64 = 100$

ASSESS

Students' answers to lesson practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 22-1 PRACTICE

11. $c^2 = 52$

12. $c^2 = 109$

13. $c^2 = 50$

14. When you square the length of the hypotenuse of a right triangle, it equals the sum of the squares of the lengths of the legs.

15. The triangle Riley drew is not a right triangle. This triangle does not follow $a^2 + b^2 = c^2$. The square of the length of the hypotenuse is 36. The sum of the squares of the legs is $3^2 + 5^2 = 9 + 25 = 34$. 34 does not equal 36, so this is not a right triangle.

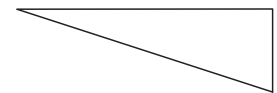
ADAPT

Check students' work to be sure they are comfortable writing equations based on the Pythagorean Theorem. If students need additional practice with this skill, assign problems from the Activity Practice. You might also assign students sets of leg lengths for a right triangle in centimeters and have them draw the legs on centimeter grid paper. Then have students complete the triangles by drawing the hypotenuses. Make sure that the leg lengths result in whole-number hypotenuse lengths (Pythagorean triples). Have students measure the hypotenuses (to the nearest centimeter) and relate the measured lengths to the leg lengths using the Pythagorean Theorem equation.

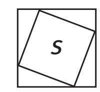
My Notes

Check Your Understanding

8. Label this triangle using *a* for leg 1, *b* for leg 2, and *c* for the hypotenuse:



Use the figure to answer Items 9–10.



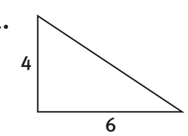
9. If the right triangle used to make the figure has leg lengths of 6 units and 8 units, what is the area of the inner square, *S*?

10. Write an equation in the form $a^2 + b^2 = c^2$ for the figure.

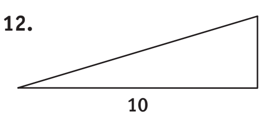
LESSON 22-1 PRACTICE

Find c^2 for the following right triangles.

11.



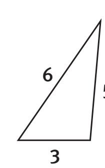
12.



13.

14. What does the Pythagorean Theorem state? Explain in your own words.

15. **Construct viable arguments.** Riley drew a triangle with the following dimensions:



Is this triangle a right triangle? Explain your reasoning.

Lesson 22-2
Pythagorean Theorem: Missing Lengths

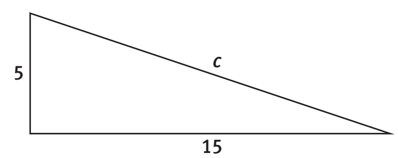
My Notes

Learning Targets:
- Investigate the Pythagorean Theorem.
- Find missing side lengths of right triangles using the Pythagorean Theorem.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Visualization, Look for a Pattern, Critique Reasoning, Sharing and Responding

The Pythagorean Theorem states that the sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse. This relationship can be used to determine the missing length of a side of a right triangle when you are given two lengths.

Example A



Find the length of the hypotenuse, c.

Step 1: Substitute the given lengths into the equation: $a^2 + b^2 = c^2$.
$$5^2 + 15^2 = c^2$$

Step 2: Square the lengths and add.
$$25 + 225 = c^2$$
$$250 = c^2$$

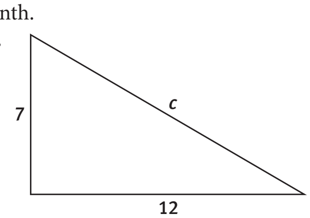
Step 3: Find the square root to solve for c. Since 250 is not a perfect square, round to the nearest tenth when finding the square root.
$$250 = c^2$$
$$15.8 = c$$

Solution: The length of the hypotenuse, c, is 15.8.

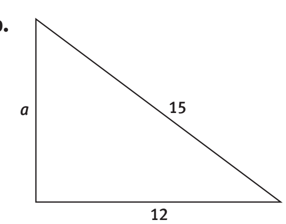
Try These A

Use the Pythagorean Theorem to find the unknown length to the nearest tenth.

a.



b.



ACTIVITY 22 Continued

Lesson 22-2

PLAN

Pacing: 1–2 class periods

Chunking the Lesson

Example A #1–2
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity
Give students a few minutes to write an explanation of the Pythagorean Theorem in their own words. Ask them to include a sketch with their statement. Take a few minutes to debrief students' work with the class, as this will serve to activate prior knowledge and lead naturally into the content of this lesson.

Example A Marking the Text, Visualization, Think-Pair-Share, Debriefing Use this example to discuss reasonableness of answers with the class. It is important for students to understand that the hypotenuse is the longest side of a right triangle. Had the answer to the problem in Example A been anything less than 15, this would not have been a reasonable answer. Some students may forget to take the final square root when solving this type of problem. In this case, the resulting answer would be $c = 250$. Again, students should recognize that this is not reasonable in a triangle whose other side lengths are 5 and 15.

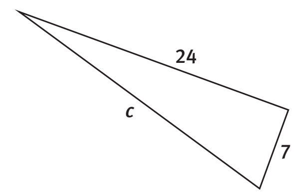
Try These A

a. $c = 13.9$
b. $a = 9$
c. $c = 25$
d. $b = 13.4$
e. hypotenuse $= 11.7$

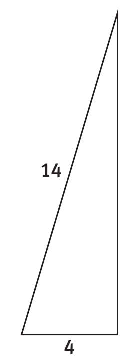
1–2 Predict and Confirm, Visualization, Look for a Pattern, Critique Reasoning, Sharing and Responding Students now have the tools to revisit the scenario that was presented on the first page of the activity (Lesson 22-1) and check whether their prediction was correct. Note that the right triangle in this problem is a 5-12-13 triangle. Students will learn more about whole numbers that satisfy the Pythagorean Theorem when they explore Pythagorean triples in Activity 24.

Lesson 22-2
Pythagorean Theorem: Missing Lengths

c.

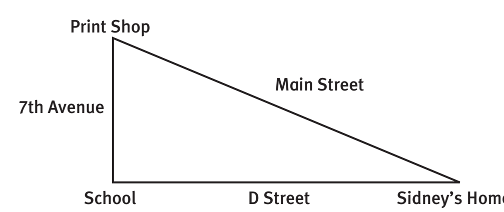


d.



e. Leg 1 = 6
Leg 2 = 10

1. Now that you know the relationship of the lengths of the three sides of any right triangle, you can figure out whether Sidney will make it to the print shop before it closes using the Pythagorean Theorem. Recall that Sidney leaves his house at 3:45 P.M. to try to make it to the print shop before 4:00 P.M. He starts biking down Main Street to the print shop. As he is pedaling, he wonders how far it is to the print shop. His house is 12 blocks away from the school and the print shop is five blocks away from the school. He can travel, at the most, one block per minute on his bike.



a. How many blocks is it from the school to the print shop?
5 blocks

b. How many blocks is it from the school to Sidney's home?
12 blocks

c. How many block lengths down Main Street will Sidney have to bike to get to the print shop?
13 blocks

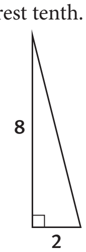
Lesson 22-2
Pythagorean Theorem: Missing Lengths

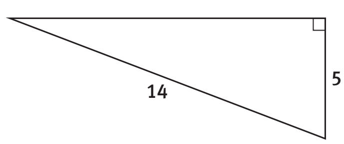
d. **Model with mathematics.** Can Sidney make it to the print shop on time? Explain your reasoning.
It is possible for Sidney to make it to the print shop on time since he needs to travel the length of 13 blocks. He can travel, at most, one block per minute, but he has 15 minutes before the print shop closes.

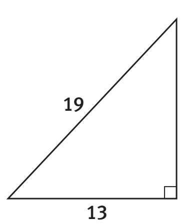
2. When you used the Pythagorean Theorem to find the distance from Sidney's house to the print shop, the formula gave you the square of the distance. What did you have to do to get the actual distance?
To determine the actual distance, I had to find the square root of the sum of 25 and 144.

Check Your Understanding

Use the Pythagorean Theorem to find the unknown length to the nearest tenth.

3.


4.


5.


My Notes

CONNECT TO AP

The Pythagorean Theorem is fundamental to the development of many more advanced mathematical topics such as the distance formula, complex numbers, and arc length of a curve.

CONNECT TO AP

The Pythagorean Theorem is one of the most useful and important relationships for students to understand and be able to use as they continue their study of mathematics. At the middle school level, students use this relationship to find the third side of a right triangle when two sides are known. Beyond middle school, topics that can trace their roots directly back to the Pythagorean Theorem include: the development of the distance formula and the equation of a circle in coordinate geometry; the circular definitions of the trigonometric functions; polar coordinates; complex numbers; calculus topics such as the derivatives of inverse trigonometric functions and finding the arc length of a curve; the magnitude of a vector; and three-dimensional geometry.

Check Your Understanding

These items provide a straightforward formative assessment of students' ability to apply the Pythagorean Theorem to a variety of right triangles. Debrief these items by asking students to summarize the main steps that they used to find the unknown side lengths in the right triangles. This will provide structure and support to students who have not yet mastered this skill.

Answers

3. 8.2
4. 13.1
5. 13.9

ASSESS

Students' answers to lesson practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 22-2 PRACTICE

6. The relationship $a^2 + b^2 = c^2$ can be used to find the missing length. Use the two lengths that are given and solve for the missing length.

7. hypotenuse = 9.5

8. 7.8 miles

9. 5.3 feet

10. I agree because this isosceles triangle is also a right triangle. The missing length can be found using $a^2 + b^2 = c^2$ and is 7.1.

ADAPT

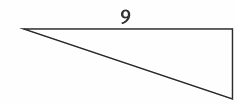
Students should be able to solve basic problems using the Pythagorean Theorem. Students who need more practice will have many more opportunities to solve such problems in the Activity Practice as well as in the next activity.

My Notes

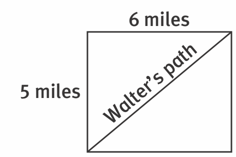
LESSON 22-2 PRACTICE

6. Explain in your own words how the Pythagorean Theorem can be used to find a missing length of a right triangle.

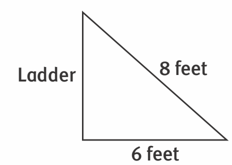
7. Find the length of the hypotenuse in this right triangle:



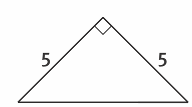
8. Walter is riding his bike across a park as shown. How far does he travel?



9. A playground slide measures 8 feet long. The slide ends 6 feet from the ladder. What is the length of the ladder?



10. **Critique the reasoning of others.** Shanti says she can use the Pythagorean Theorem to find the missing length on this isosceles triangle. Do you agree with her reasoning? Explain.

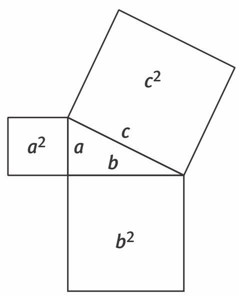


ACTIVITY 22 PRACTICE

Write your answers on notebook paper.
Show your work.

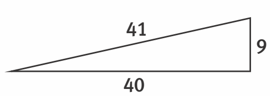
Lesson 22-1

1. This diagram shows the squares of the lengths of the sides of a right triangle. Copy the table and refer to the diagram to complete.

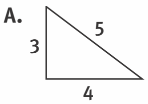


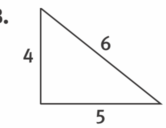
Case	a	b	c	a^2	b^2	c^2
1	3	4	5			
2				81	144	225
3				36	64	100
4	8	15	17			

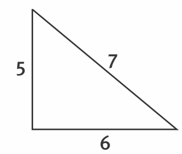
2. Find c^2 given a triangle whose legs measure 5 units and 8 units.
3. Write the Pythagorean Theorem equation for this right triangle.

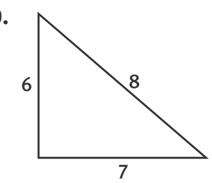


4. If you know the lengths of the sides of a triangle, how might you use the Pythagorean Theorem to tell if the triangle is or is not a right triangle?
5. Which of the following is a right triangle?

A.


B.


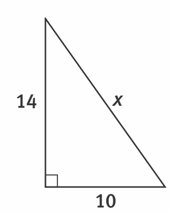
C.


D.


6. Roman says the Pythagorean Theorem applies to all triangles. Do you agree with his statement? Explain your reasoning.

Lesson 22-2

7. Find x in the triangle below.



ACTIVITY 22 Continued

ACTIVITY PRACTICE

1.

Case	a	b	c	a^2	b^2	c^2
1	3	4	5	9	16	25
2	9	12	15	81	144	225
3	6	8	10	36	64	100
4	8	15	17	64	225	289

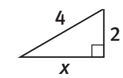
2. $c^2 = 89$
3. $9^2 + 40^2 = 41^2$
4. When you know the lengths of the sides of a triangle, then you can substitute the lengths in the Pythagorean Theorem using the longest length for c and see whether the equation is true. If it is, the triangle is a right triangle; if not, it is not.
5. A
6. Answers will vary. Roman is not correct in stating the Pythagorean Theorem applies to all triangles. It is only true for right triangles.
7. $x \approx 17.2$

8. $x \approx 3.5$
9. $x = 26$
10. 25 feet
11. C
12. D
13. 14.1 m
14. 15.5 inches
15. 10 blocks
16. 13.4 meters
17. Check dimensions of the triangle to verify that a right triangle was drawn.

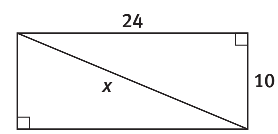
ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.

8. Find x in the triangle below.



9. Find x in the triangle below.



10. A painter uses a ladder to reach a second-story window on the house she is painting. The bottom of the window is 20 feet above the ground. The foot of the ladder is 15 feet from the house. How long is the ladder?

11. Which length is the greatest?
A. the diagonal of a square with 4-in. sides
B. the hypotenuse of a right triangle with legs of length 3 in. and 4 in.
C. the diagonal of a rectangle with sides of 5 in. and 12 in.
D. the perimeter of a square with side lengths of 1 in.

12. A hiker leaves her camp in the morning. How far is she from camp after walking 9 miles west and then 10 miles north?
A. 19 miles
B. 4.4 miles
C. 181 miles
D. 13.5 miles

13. A brick walkway forms the diagonal of a square playground. The walkway is 20 m long. To the nearest tenth of a meter, how long is one side of the playground?

14. The screen size of a television is measured along the diagonal of the screen from one corner to another. If a television has a length of 28 inches and a diagonal that measures 32 inches, what is the height of the television set to the nearest tenth?

15. Tim's cousin lives 8 blocks due south of his house. His grandmother lives 6 blocks due east of him. What is the distance in blocks from Tim's cousin's house to Tim's grandmother's house?

16. A rectangular garden is 6 meters wide and 12 meters long. Sean wants to build a walkway that goes along the diagonal of the garden. How long will the walkway be?

MATHEMATICAL PRACTICES
Attend to Precision

17. Use grid paper to draw a right triangle. Count the units for the legs. Calculate the length of the hypotenuse using the Pythagorean Theorem.

Applying the Pythagorean Theorem

ACTIVITY 23

Diamond in the Rough

Lesson 23-1 The Pythagorean Theorem in Two and Three Dimensions

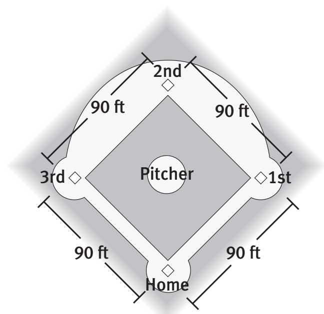
Learning Targets:

- Apply the Pythagorean Theorem to solve problems in two dimensions.
- Apply the Pythagorean Theorem to solve problems in three dimensions.

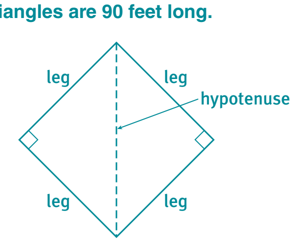
SUGGESTED LEARNING STRATEGIES: Marking the Text, Close Reading, Paraphrasing, Identify a Subtask, Think-Pair-Share

Cameron is a catcher trying out for the school baseball team. He has played baseball in the community and is able to easily throw the ball from home plate to second base to throw out a runner trying to steal second base. However, the school baseball diamond is a regulation-size field and larger than the field he is accustomed to.

The distance between each consecutive base on a regulation baseball diamond is 90 feet and the baselines are perpendicular. The imaginary line from home plate to second base divides the baseball diamond into two right triangles. There is a relationship between the lengths of the three sides of any right triangle that might be helpful for determining if Cameron can throw across a regulation baseball diamond.



1. Sketch a diagram of a regulation baseball diamond showing the baselines and the imaginary line from home plate to second base. Identify and label the hypotenuse and legs of any right triangles. What are the lengths of the legs of the triangles?
   The legs of the triangles are 90 feet long.



My Notes

DISCUSSION GROUP TIPS

In discussion groups, read the text carefully to clarify the meaning of math terms and other vocabulary.

Common Core State Standards for Activity 23

8.G.B.6 Explain a proof of the Pythagorean Theorem and its converse.

8.G.B.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

8.G.B.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

ACTIVITY 23

Guided

Activity Standards Focus

In this activity, students apply the Pythagorean Theorem to solve problems in two and three dimensions. Students also use the Pythagorean Theorem on the coordinate plane in order to find the distance between a pair of given points.

Lesson 23-1

PLAN

Pacing: 1–2 class periods

Chunking the Lesson

#1 #2–6

Check Your Understanding

#9–12

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Activate students' prior knowledge by asking them to find the length of the hypotenuse of a right triangle with legs that are 9 cm and 12 cm long. Ask students to share their answers and explain how they found the required length. Be sure students understand that the Pythagorean Theorem is the underlying relationship that allows them to calculate the unknown side length. Explain that this lesson will focus on new applications of the theorem.

Teacher to Teacher

Some students may be unfamiliar with baseball and its regulations. Using learning strategies, like Shared Reading and Marking the Text, to read the introduction will allow all students to begin the activity with the necessary background information.

1 Marking the Text, Close Reading, Paraphrasing, Create Representations, Think-Pair-Share, Debriefing Have students share their diagrams and compare them for accuracy before they continue the activity. Be sure students recognize that drawing a line from home plate to second base creates two right triangles with a common hypotenuse.

2–6 Identify a Subtask, Think-Pair-Share, Debriefing, Discussion Groups In Item 3, students may answer either yes or no. It is possible to find an approximate square root without a calculator, but it is more difficult without technology. Students may compare methods that they are familiar with for estimating the square root. In Item 4, students may need to be reminded of the definitions of rational and irrational numbers.

Teacher to Teacher

It may be interesting for students to compare the size of the softball diamond to that of the baseball diamond. There could be many opinions about why softball diamonds are smaller. Students might consider the size and weight of the ball involved as well.

ELL Support

To support students in acquiring and using new language, especially as discussions focus on increasingly more challenging concepts, provide linguistic support through translations of key terms and other language. Group students carefully to ensure participation of all group members in class discussions.

Lesson 23-1

The Pythagorean Theorem in Two and Three Dimensions

MATH TIP

The Pythagorean Theorem states that the square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the legs of the triangle.

MATH TIP

If you take the square root of a number that is not a perfect square, the result is a decimal number that does not terminate or repeat and is therefore an irrational number.

2. Write an equation that can be used to find the distance from home plate to second base.

$c^2 = 90^2 + 90^2$

3. Use appropriate tools strategically. Can the distance from home plate to second base be found without a calculator? Why or why not?

Sample answer: It can be done, but it is much simpler to do with a calculator, as $\sqrt{16,200}$ is a large number.

4. Is this value from Item 3 a rational or irrational number? Using a calculator, give the approximate length of the distance from home plate to second base.

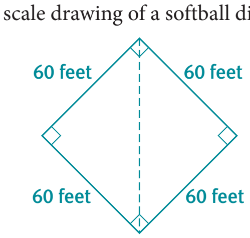
Since 16,200 is not a perfect square, the value is irrational (it is a nonterminating, nonrepeating decimal). $\sqrt{16,200} \approx 127.28$ feet

5. If Cameron can throw the baseball 130 feet, will he be able to consistently throw out a runner trying to steal second base? Explain your reasoning.

Sample explanation: Yes. Since Cameron can throw the ball 130 feet, and the throw he has to make is less than that, he will be able to consistently throw out a runner trying to steal second.

6. On a regulation softball diamond, the distance between consecutive bases is 60 feet and the baselines are perpendicular.

a. Sketch and label a scale drawing of a softball diamond.



b. Use your sketch to approximate the distance from home plate to second base on a softball field. Show all your work.

$c^2 = 60^2 + 60^2$; $c \approx 84.85$ feet

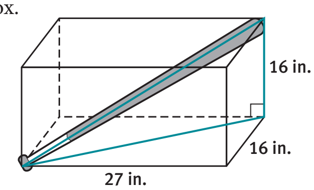
My Notes

Check Your Understanding

7. A rectangular garden is 6 meters wide and 12 meters long. Sean wants to build a walkway that goes through the diagonal of the garden. How long will the walkway be? Round to the nearest hundredth.
8. A rectangular computer screen has a diagonal length of 21 inches. The screen is 11 inches wide. To the nearest tenth of an inch, what is the length of the screen?

During summer vacation, Cameron's parents take him to see his favorite baseball team play. On their last day of vacation, he discovers that he will not be able to carry the autographed bat that he won home on the plane. His dad suggests that he speak to the concierge at the hotel about options for shipping the bat home.

The concierge has only one box that he thinks might be long enough. After measuring the dimensions of the box to be 16 in. × 16 in. × 27 in., the concierge apologizes for not having a box long enough for the 34 inch bat. Cameron thinks he might still be able to use the box. His idea is to put the bat in the box at an angle as shown in the diagram below. He wonders if the bat will fit in the box.



CONNECT TO TRAVEL

In a hotel, a concierge is a person who helps guests with various tasks ranging from restaurant reservations to travel plans.

9. The diagonal of the box is the hypotenuse of a right triangle. Outline this triangle in the diagram above.

10. What are the lengths of the legs of this right triangle? Show any work needed to find these lengths.
    The legs of this triangle are 16 inches and 31.38 inches.
    $c^2 = 16^2 + 27^2$; $c \approx 31.38$ inches

11. Find the length of the diagonal of the box. Show any necessary calculations.
    ≈ 35.2 inches; $c^2 = 16^2 + 31.38^2$

12. Will Cameron be able to use the box to ship his bat? Justify your response.
    Yes. The bat is 34 inches long. Since the diagonal of the box is 35.2 inches, the bat will fit if he places it diagonally in the box.

Check Your Understanding

Debrief students' answers to these items for a formative assessment of students' ability to interpret a word problem, sketch an appropriate diagram, and use the Pythagorean Theorem to find an unknown length.

Answers

7. 13.42 meters
8. 17.9 inches

9–12 Visualization, Create Representations, Think-Pair-Share, Group Presentation Have students share the diagonal of the box that they drew so that they all have the same information before continuing to work on the problem. Students may struggle to understand that these questions require using the Pythagorean Theorem twice. It is first used in Item 10 to find the hypotenuse of the right triangle on the bottom of the box with legs of 27 inches and 16 inches. It is used a second time in Item 11 to find the length of the diagonal of the box.

Differentiating Instruction

Support Visual and kinesthetic learners will benefit from modeling the problem with an actual cardboard box, such as a shoebox.

Extend You many want to ask students to develop a general formula for finding the length of the diagonal of a box with length ℓ, width w, and height h. The formula is $d = \sqrt{\ell^2 + w^2 + h^2}$.

Check Your Understanding

Debrief these items with the class to ensure that students can apply the Pythagorean Theorem in three dimensions. This also serves as an assessment of students' ability to use their calculators to help them determine length to the nearest tenth of an inch.

Answers

13. 23.3 inches
14. 22.9 inches

ASSESS

Students' answers to lesson practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 23-1 PRACTICE

15. 14.97 centimeters
16. 5.7 meters
17. 37.7 feet
18. 8

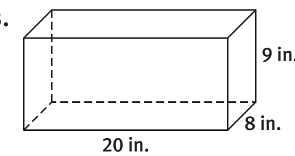
ADAPT

Check students' work to be sure they can apply the Pythagorean Theorem in a variety of real-world settings. Note that most of the problems in this lesson require drawing an auxiliary line in a figure, such as a diagonal of a rectangle or rectangular prism. Check students' sketches to be sure they know when it is necessary to draw such a line.

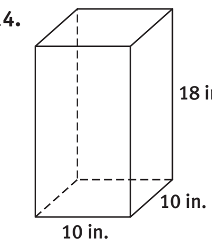
Check Your Understanding

Cameron brought some collapsible fishing rods on his vacation. Find the length of the longest fishing rod that he can fit in each of the boxes shown below. Round to the nearest tenth.

13.



14.

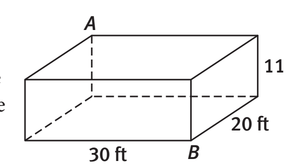


LESSON 23-1 PRACTICE

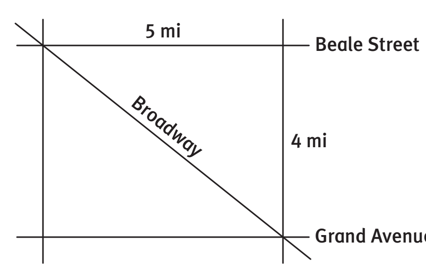
15. A rectangular photograph has a diagonal length of 18 centimeters. The photograph is 10 centimeters wide. What is the length of the photograph to the nearest hundredth of a centimeter?

16. A square window is 2 meters long on each side. To protect the window during a storm, Marisol plans to put a strip of duct tape along each diagonal of the window. To the nearest tenth of a meter, what is the total length of duct tape Marisol will need?

17. The figure shows the dimensions of a classroom. What is the distance that a moth travels if it flies in a straight line from point *A* to point *B*? Round to the nearest tenth.



18. **Make sense of problems.** A city employee is organizing a race down Broadway, from Beale Street to Grand Avenue. There will be a water station at the beginning and end of the race. There will also be water stations along the route, with no more than one mile between stations. What is the minimum number of water stations for this race?



Lesson 23-2
The Pythagorean Theorem and the Coordinate Plane

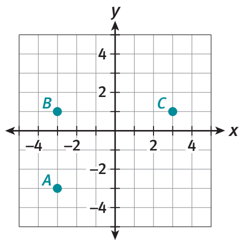
Learning Targets:
- Apply the Pythagorean Theorem to right triangles on the coordinate plane.
- Find the distance between points on the coordinate plane.

SUGGESTED LEARNING STRATEGIES: Create Representations, Think-Pair-Share, Identify a Subtask, Group Presentation

Effective baserunning is one of the essential skills that every baseball player must master. Cameron's coach spends a lot of time working with his players to help them be successful when running bases.

Part of the coach's baserunning training involves drills. A drill is an exercise for teaching a particular skill. To teach baserunning, the coach sets up a coordinate plane on the field, as shown below. Each unit of the coordinate plane represents 10 feet. The coach places bases on the coordinate plane and has players sprint or slide between the bases in various patterns.

For the first drill, the coach places bases at $A(-3, -3)$, $B(-3, 1)$, and $C(3, 1)$. The coach has the players run from A to B to C and then run and slide back to A as quickly as possible.



My Notes

ACTIVITY 23 Continued

Lesson 23-2

PLAN

Pacing: 1–2 class periods

Chunking the Lesson

#1 #2–4
Check Your Understanding
Example A
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity
Ask students to draw a horizontal line segment and a vertical line segment on a coordinate plane. Then have them determine the lengths of the segments. Have students share their work and describe their methods. Be sure students realize that they can determine these lengths simply by counting units. Tell the class they will use this skill in this lesson.

Introduction Shared Reading Have students read the introductory paragraphs and mark the text for relevant information.

1 Marking the Text, Close Reading, Create Representations Students should be familiar with plotting points on the coordinate plane. However, students might need to be led to the discovery that when the given points are connected, they form a right triangle.

Teacher to Teacher

A key observation for this lesson is that horizontal and vertical lines intersect to form a right angle. This is why the given points form a right triangle and why it is appropriate to apply the Pythagorean Theorem to find the distance between two points. Although the Distance Formula is not formally introduced, the underlying concepts of the Distance Formula are explored in this lesson, preparing students to use the formula in future courses.

2–4 Identify a Subtask, Think-Pair-Share, Debriefing, Group Presentation Students should have the opportunity to discuss in their groups the difference between calculating the distance in Item 2 and calculating the distance between C and A in Item 3. Justifications in Item 4 can vary, but using the Pythagorean Theorem to find the distance should be discussed during debriefing.

Check Your Understanding

These items serve as a formative assessment of students' understanding of right triangles on the coordinate plane. As you debrief students' work, check that they form right triangles from the given points and that they understand how to use the Pythagorean Theorem to find the required lengths.

Answers

5. 5 units
6. 8.1 units

My Notes

Lesson 23-2
The Pythagorean Theorem and the Coordinate Plane

Cameron's coach wants to know approximately how far the players will run and slide as they go from base C back to base A.

1. Plot and label the bases on the coordinate plane on the previous page.
2. What is the distance between bases A and B? What is the distance between the bases B and C?
   The distance between bases *A* and *B* is 4 units or 40 feet.
   The distance between bases *B* and *C* is 6 units or 60 feet.
3. Can you use the same method that you used in Item 2 to find the distance between bases C and A? Why or why not?
   No. Sample answer: I can count vertical and horizontal distances but the distance between *C* and *A* is neither horizontal nor vertical.
4. **Make use of structure.** Calculate the shortest distance between bases C and A. Explain and justify your method.
   Sample explanation: Connect the bases to make a right triangle. Then use the Pythagorean Theorem to find the distance between *C* and *A*.
   (distance between *C* and *A*)$^2 = 40^2 + 60^2$
   (distance between *C* and *A*)$^2 = 5{,}200$
   distance between *C* and *A* ≈ 72.1 feet

Check Your Understanding

Use the My Notes column on this page to plot the points to find the length of the hypotenuse in each right triangle. Round to the nearest tenth, if necessary.

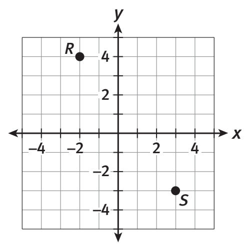
5. $D(-3, 0)$, $E(0, 0)$, $F(0, 4)$
6. $G(-4, 2)$, $H(3, 2)$, $J(3, -2)$

Lesson 23-2
The Pythagorean Theorem and the Coordinate Plane

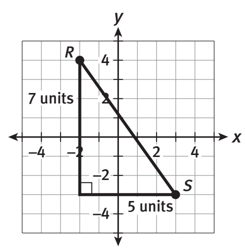
Example A

Find the distance between $R(-2, 4)$ and $S(3, -3)$.

Step 1: Plot the points.



Step 2: Draw a right triangle. Find the length of the legs.



Step 3: Use the Pythagorean Theorem to find the length of the hypotenuse.

$$RS^2 = 7^2 + 5^2$$
$$RS^2 = 74$$
$$RS \approx 8.6 \text{ units}$$

Solution: The distance between the points is approximately 8.6 units.

Try These A

Find the distance between each pair of points. Round to the nearest tenth, if necessary.

a. $(-1, 3)$ and $(2, 1)$

b. $(6, 2)$ and $(-4, -2)$

My Notes

WRITING MATH

Write $\overline{RS}$ to represent the line segment with endpoints R and S. Write RS to represent the length of the line segment or the distance between R and S.

Example A Identify a Subtask, Create Representations, Think-Pair-Share This example is similar to the work students have done so far in this lesson, except students are now only given two points to work with. Students see that when those two points become the endpoints of the hypotenuse of a right triangle, the distance between the points can be found by applying the Pythagorean Theorem and finding the length of the hypotenuse.

Teacher to Teacher

This example offers a good opportunity to discuss reasonableness of answers. Given that the longer leg of the right triangle in Step 2 is 7 units long, it should seem reasonable to students that the hypotenuse is 8.1 units long. Be sure students are able to give examples of answers that would not have been reasonable. For instance, an answer of 3.2 units would have been too short and an answer of 73.9 units would have been much too long.

Try These A

a. 3.6 units

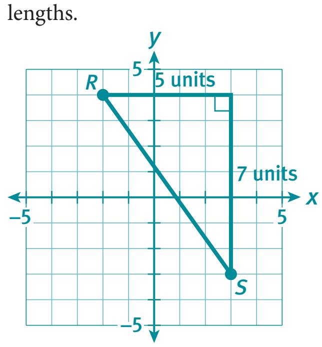
b. 10.8 units

Check Your Understanding

Debrief students' answers to these items to ensure that students have a deep understanding of Example A.

Answers

7. This right triangle gives the same result because its legs have the same lengths.



8. Answers may vary. The hypotenuse is the longest side of a right triangle. The answer is reasonable, since 8.1 units is a bit greater than the length of either of the legs.

ASSESS

Students' answers to lesson practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 23-2 PRACTICE

9. 5.7 units
10. 8.6 units
11. 5.1 units
12. 9.4 units
13. 206 feet
14. No; the total distance is a bit more than 7 miles, so the total time for the trip will be at least 35 minutes.

ADAPT

Check students' work to ensure that they can apply the Pythagorean Theorem on the coordinate plane. If students need additional practice, have them plot pairs of points of their own choosing on a coordinate plane and then ask them to find the distance between the points. You can also assign problems from the Activity Practice for additional work with this skill.

My Notes

Check Your Understanding

7. Carlos looked at the figure in Example A and said that there is a different way to draw a right triangle that has $\overline{RS}$ as its hypotenuse. Draw a figure to show what Carlos means. Does this right triangle give the same result? Explain.

8. Use what you know about triangles to explain why the answer to Example A is reasonable.

LESSON 23-2 PRACTICE

Find the length of the hypotenuse in each right triangle. Round to the nearest tenth, if necessary.

9. $P(1, 3)$, $Q(5, 3)$, $R(5, -1)$

10. $L(-3, 2)$, $M(-3, -3)$, $N(4, -3)$

Find the distance between each pair of points. Round to the nearest tenth, if necessary.

11. $(-2, -2)$ and $(-1, 3)$

12. $(1, -3)$ and $(6, 5)$

13. During a drill, Cameron's coach has players sprint from $J(3, 2)$ to $K(-4, 2)$ to $L(-4, -3)$ and back to J. Each unit of the coordinate plane represents 10 feet. To the nearest foot, what is the total distance players sprint during this drill?

14. Attend to precision. On a map of Ayana's town, the library is located at $(-5, -3)$ and the middle school is located at $(0, 2)$. Each unit of the map represents one mile. Ayana wants to bike from the middle school to the library. She knows that it takes her about 5 minutes to bike one mile. Will she be able to make the trip in less than half an hour? Explain.

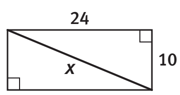
ACTIVITY 23 PRACTICE

Write your answers on notebook paper. Show your work.

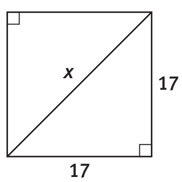
Lesson 23-1

Find x in each of the following figures. Round to the nearest tenth, if necessary.

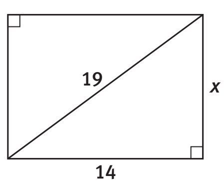
1.



2.



3.



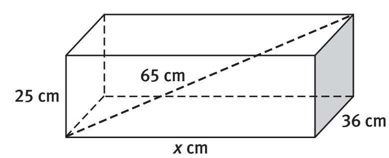
4. Riyo wants to place a string of lights across the ceiling of her bedroom. The room is a rectangle that is 18 feet long and 15 feet wide. Her string of lights is 20 feet long. Will the string of lights be long enough to hang diagonally from one corner of the ceiling to the other? Explain.

5. Which of the following lengths is the greatest?
A. the diagonal of a square with 4-in. sides
B. the hypotenuse of a right triangle with legs of length 3 in. and 4 in.
C. the diagonal of a rectangle with sides of length 5 in. and 12 in.
D. the perimeter of a square with side lengths of 1 in.

6. Gary has a rectangular painting that is 21 inches wide and 36 inches long. He wants to place wire in the shape of an X on the back of the painting along its diagonals so that he can hang the painting on the wall. Which is the best estimate of the total amount of wire Gary will need?
A. 42 inches **B.** 57 inches
C. 84 inches **D.** 114 inches

7. What is the length of the longest fishing pole that will fit in a box with dimensions 18 in., 24 in., and 16 in.?

8. The box below has dimensions 25 cm, 36 cm, and x cm. The diagonal shown has a length of 65 cm. Find the value of x. Round to the nearest tenth, if necessary.



9. A brick walkway forms the diagonal of a square playground. The walkway is 20 m long. To the nearest tenth of a meter, how long is one side of the playground?

10. A television set's screen size is measured along the diagonal of the screen from one corner to another. If a television screen has a length of 28 inches and a diagonal that measures 32 inches, what is the height of the screen to the nearest tenth?

11. A rectangle has sides of length p and q. Which expression represents the length of the diagonal of the rectangle?
A. $2(p+q)$ **B.** p^2+q^2
C. $\sqrt{p+q}$ **D.** $\sqrt{p^2+q^2}$

ACTIVITY PRACTICE

1. 26
2. 24.0
3. 12.8
4. No; the diagonal of the room is about 23.4 feet long.
5. C
6. C
7. 34 inches
8. 48 inches
9. 14.1 meters
10. 15.5 inches
11. D

12. 3.6 units
13. 4.2 units
14. 5 units
15. 6 units
16. 10 units
17. 12.8 units
18. D
19. B
20. 4.5 units
21. a. Answers may vary. (4, 5) or (4, −3)
    b. Yes; the base could be located at (4, 5) or (4, −3).
    c. 50 feet; this distance does not depend upon which of the possible locations for base *C* the coach chooses because the lengths of the legs of the right triangle are the same in either case.
22. 7.1 miles
23. about 0.7 mile
24. No; the distance is about 10.6 miles. At the rate of 4 miles per hour, he will only be able to walk 8 miles in 2 hours.
25. a. 5 units
    b. Answers may vary. (0, 5), (−5, 0), (−3, −4), and (4, −3)
    c. Check students' graphs.
    d. circle

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.

Lesson 23-2

For Items 12–17, find the distance between each pair of points. Round to the nearest tenth, if necessary.

12. (0, 0) and (3, 2)

13. (−3, −1) and (0, 2)

14. (−1, 1) and (3, −2)

15. (2, −1) and (2, 5)

16. (6, −2) and (−2, 4)

17. (−3, −5) and (5, 5)

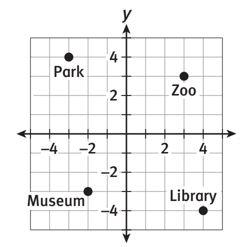
18. Which is the best estimate of the distance between the points *A*(4, −5) and *B*(−2, 1)?
A. 7 units **B.** 7.5 units
C. 8 units **D.** 8.5 units

19. Which point lies the farthest from the origin?
A. (−6, 0) **B.** (−3, 8)
C. (5, 1) **D.** (−4, −3)

20. How far from the origin is the point (−2, −4)? Round to the nearest tenth, if necessary.

21. For a baserunning drill, a coach places bases at *A*(1, 1) and *B*(4, 1), where each unit of the coordinate plane represents 10 feet. The coach wants to locate base *C* so that the distance from *B* to *C* is 40 feet and so that the three bases form a right triangle.
a. What is a possible location for base *C*?
b. Is there more than one possibility for the location of base *C*? Explain.
c. What is the distance from base *A* to base *C*? Does this distance depend upon which of the possible locations for base *C* the coach chooses? Justify your response.

The coordinate plane shows a map of Elmville. Each unit of the coordinate plane represents one mile. Use the map for Items 22–24.



22. What is the distance from the zoo to the library? Round to the nearest tenth of a mile.

23. Assuming it is possible to walk between locations in a straight line, how much longer is it to walk from the museum to the zoo than to walk from the museum to the park?

24. Donnell plans to walk from the park to the library along a straight route. If he walks at 4 miles per hour, can he complete the walk in less than 2 hours? Explain.

MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

25. Consider the points *A*(5, 0), *B*(−3, 4), and *C*(−4, 3).
a. Find the distance of each point from the origin.
b. Give the coordinates of four additional points that are this same distance from the origin.
c. Plot the given points and the points you named in part b.
d. Suppose you continued to plot points that are this same distance from the origin. What geometric figure would the points begin to form?

The Converse of the Pythagorean Theorem

ACTIVITY 24

Paper Clip Chains

Lesson 24-1 The Converse of the Pythagorean Theorem

My Notes

Learning Targets:

- Explain the converse of the Pythagorean Theorem.
- Verify whether a triangle with given side lengths is a right triangle.

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Visualization, Discussion Group, Create Representations, Note Taking

It is believed that the Pythagorean Theorem was applied in the building of the pyramids and the establishment of land boundaries in ancient Egypt. Egyptian surveyors, known as rope stretchers, applied the theorem to reestablish property lines after the annual flooding of the Nile. They created right angles by forming right triangles using long ropes with 13 equally spaced knots tied in them to create 12 equal sections of rope.

To understand how the Egyptian rope stretchers made their right triangles, complete the following items.

1. Use 12 paper clips to create a right triangle like the ones the Egyptians made from rope. Draw a sketch of the triangle you created. Label the number of paper clips on each side and the location of what you believe is the right angle.
   Students should create a 3, 4, 5 right triangle. The hypotenuse should be 5 paper clips long.

2. Give reasons to support your belief that your triangle is a right triangle.
   Sample answers include choosing to justify an answer using the Pythagorean Theorem to show that $5^2 = 3^2 + 4^2$, or justifying an answer by measuring the angle with the corner of a sheet of paper or a note card.

3. Form another triangle with the 12 paper clips, with side lengths different from your original triangle. What are the lengths of the sides? What is the best name for the triangle you made?
   Sample answers: a 4, 4, 4 equilateral triangle and a 5, 5, 2 isosceles triangle

Common Core State Standards for Activity 24

8.G.B.6 Explain a proof of the Pythagorean Theorem and its converse.

8.G.B.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

ACTIVITY 24

Investigative

Activity Standards Focus

This activity gives students a chance to investigate and apply the converse of the Pythagorean Theorem. Students also identify and create sets of whole numbers that are Pythagorean triples.

Lesson 24-1

PLAN

Materials

- paper clips
- index card

Pacing: 1 class period

Chunking the Lesson

#1–3 #4 #5–6

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Ask students to do a Quickwrite in which they write down everything they know about the Pythagorean Theorem. Take time to have volunteers share their work to solidify students' understanding of what the theorem says.

Teacher to Teacher

The Pythagorean Theorem states that if a triangle with side lengths a, b, and c is a right triangle, then $a^2 + b^2 = c^2$, where a and b are the lengths of the legs, and c is the length of the hypotenuse. The converse of the Pythagorean Theorem goes in the opposite direction. It states that if the side lengths of a triangle satisfy the relationship $a^2 + b^2 = c^2$, then the triangle is a right triangle.

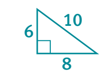
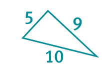
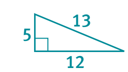
1–3 Activating Prior Knowledge, Use Manipulatives, Discussion Groups, Create Representations, Note Taking Small standard paper clips should be used for this investigation. Students should first hook the 12 paper clips together to create a chain. Where the paper clips meet represents the knots on the rope and the paper clips themselves represent the spaces between the knots on the rope.

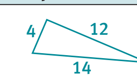
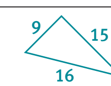
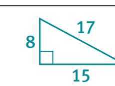
In Item 3, students should be able to show that their triangle is not a right triangle and tell what type of triangle it is. Debriefing the types of triangles students create is a good review of geometry concepts. Students may classify the triangles by side lengths, by angle measures, or both. This information can be posted in the classroom as review.

4 Use Manipulatives, Graphic Organizer, Visualization, Look for a Pattern, Create Representations, Debriefing Students will need to create chains, each with the specified number of paper clips, and form triangles with the correct number of paper clips on each side. Students should use the right angle at the corner of an index card to determine if the triangles are right triangles.

My Notes

4. Use paper clips to create triangles having the given side lengths. Use a corner of an index card to test the largest angle of each triangle for a right angle and predict whether or not the given triangles are right triangles. Draw and label a sketch of each triangle formed.

Triangle Side Lengths	Right Triangle?	Pictorial Representation
6, 8, 10	Yes	
5, 9, 10	No	
5, 12, 13	Yes	

Triangle Side Lengths	Right Triangle?	Pictorial Representation
4, 12, 14	No	
9, 15, 16	No	
8, 15, 17	Yes	

5. Using $c^2 = a^2 + b^2$, where c is the longest side, support your predictions for each triangle in Item 4. Use the chart below to show your work.
Answers in the Prediction Correct? column may vary.

Triangle Side Lengths	c^2	(?) $=$ or $\neq$	$a^2 + b^2$	Prediction Correct?
6, 8, 10	100	$=$	36 + 64	
5, 9, 10	100	$\neq$	25 + 81	
5, 12, 13	169	$=$	25 + 144	
4, 12, 14	196	$\neq$	16 + 144	
9, 15, 16	256	$\neq$	81 + 225	
8, 15, 17	289	$=$	64 + 225	

My Notes

READING MATH

The symbol $=$ is read "is equal to" and the symbol $\neq$ is read "is not equal to."

5–6 Graphic Organizer, Look for a Pattern, Think-Pair-Share, Debriefing In Item 5, students should begin to notice a pattern. Specifically, triangles whose side lengths satisfy the relationship $a^2 + b^2 = c^2$ are right triangles. Students state this observation in Item 6. This observation is the converse of the Pythagorean Theorem.

Differentiating Instruction

To help struggling students understand the difference between the Pythagorean Theorem and its converse, work with students to create a graphic organizer like the one below.

Pythagorean Theorem

right triangle

$a^2 + b^2 = c^2$

Converse of the Pythagorean Theorem

Check Your Understanding

Debrief students' answers to these items as a quick formative assessment to check whether students can apply the converse of the Pythagorean Theorem.

Answers

7. Yes; $25^2 = 24^2 + 7^2$
8. No; $13^2 \neq 6^2 + 12^2$

ASSESS

Students' answers to lesson practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 24-1 PRACTICE

9. No; $16^2 \neq 8^2 + 12^2$
10. Yes; $26^2 = 24^2 + 10^2$
11. No; $13^2 \neq 10^2 + 11^2$
12. No; $7^2 \neq 6^2 + 4.5^2$; change the length of the longest side to 7.5 inches because $7.5^2 = 6^2 + 4.5^2$.
13. No; $40^2 \neq 27^2 + 36^2$

ADAPT

Check students' work to ensure that they know how and when to apply the converse of the Pythagorean Theorem. If students need additional practice, have them generate sets of three whole numbers and have them decide whether the numbers could be the side lengths of a right triangle. You might have students generate the sets of numbers by rolling a 12-sided die three times.

My Notes

6. Express regularity in repeated reasoning. If the sides of a triangle satisfy the equation $c^2 = a^2 + b^2$, what can be said about the triangle? What must be true about c?
The triangle is a right triangle, and c must be the hypotenuse.

The relationship that you have just explored is called the Converse of the Pythagorean Theorem. It states that if the sum of the squares of the two shorter sides of a triangle equal the square of the longest side, then the triangle is a right triangle.

Check Your Understanding

Tell whether each set of side lengths forms a right triangle. Justify your response.

7. 7, 24, 25

8. 6, 12, 13

LESSON 24-1 PRACTICE

Tell whether each set of side lengths forms a right triangle. Justify your response.

9. 8, 12, 16

10. 10, 24, 26

11. Isabella has sticks that are 10 cm, 11 cm, and 13 cm long. Can she place the sticks together to form a right triangle? Justify your answer.

12. The triangular sail of a toy sailboat is supposed to be a right triangle. The manufacturer says the sides of the sail have lengths of 4.5 inches, 6 inches, and 7 inches. Is the sail a right triangle? If not, how could you change one of the lengths to make it a right triangle?

13. Model with mathematics. Alan made a small four-sided table for his office. The opposite sides of the table are 27 inches long and 36 inches long. If the diagonal of the table measures 40 inches, does the table have right angles at the corners? Why or why not?

Lesson 24-2
Pythagorean Triples

My Notes

Learning Targets:
- Verify whether a set of whole numbers is a Pythagorean triple.
- Use a Pythagorean triple to generate a new Pythagorean triple.

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Visualization, Discussion Group, Create Representations, Note Taking

A **Pythagorean triple** is a set of three whole numbers that satisfies the equation $c^2 = a^2 + b^2$.

1. **Make use of structure.** Choose 3 Pythagorean triples from Lesson 24-1 and list them in the first column of the table below. Multiply each Pythagorean triple by 2. Is the new set of numbers a Pythagorean triple? Repeat by multiplying each original set of numbers by 3.
   Sample answers shown below.

Pythagorean Triple	Multiply by 2	Pythagorean Triple?	Multiply by 3	Pythagorean Triple?
3, 4, 5	6, 8, 10	Yes	9, 12, 15	Yes
5, 12, 13	10, 24, 25	Yes	15, 36, 39	Yes
8, 15, 17	16, 30, 34	Yes	24, 45, 51	Yes

2. What do you notice when you multiply each value in a Pythagorean triple by a whole-number constant? Make a conjecture based on your results in the table.
   Multiplying each value in a Pythagorean triple by a whole-number constant results in a new Pythagorean triple.

ACTIVITY 24 Continued

Lesson 24-2

PLAN

Pacing: 1 class period

Chunking the Lesson

#1–2

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Give students a few minutes to look through their work from the last lesson and make a list of sets of whole numbers that could be the side lengths of a right triangle. Discuss students' results and compile a master list for the class.

1–2 Graphic Organizer, Visualization, Discussion Group, Create Representations, Note Taking, Think-Pair-Share Item 1 can be done using groups or Think-Pair-Share to save time if needed. Students should discuss solutions to this item so that they see that each set of Pythagorean triples is still a Pythagorean triple when multiplied by 2 or 3.

ELL Support

This lesson introduces the term *Pythagorean triple*. To support students' language acquisition, monitor group discussions to listen to pronunciation of new terms and students' use of terms to describe mathematical concepts. For students whose first language is not English, monitor understanding and use of new language structures. To support students in group discussions, suggest that they make notes about what they want to say, reviewing their notes to ensure that they are using the correct language structures. Encourage students to ask questions about the meaning of new expressions they hear as a part of your classroom discussion or during their group discussions.

Teacher to Teacher

A Pythagorean triple is a *primitive Pythagorean triple* if the greatest common factor of the numbers in the triple is 1. For example, 5, 12, 13 is a primitive Pythagorean triple, since the GCF of 5, 12, and 13 is 1. The triple 6, 8, 10 is *not* a primitive Pythagorean triple, since the GCF of these numbers is 2.

Check Your Understanding

Use these items as a formative assessment to be sure students know what is meant by the term *Pythagorean triple*. Item 3 can lead to an interesting discussion about the number of Pythagorean triples that exist. The discussion will help students see that a single Pythagorean triple can be used to generate infinitely many new Pythagorean triples.

Answers

3. Infinitely many; every whole-number multiple results in a new Pythagorean triple and there are infinitely many whole numbers

4. Yes; $169^2 = 65^2 + 156^2$

ASSESS

Students' answers to lesson practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 24-2 PRACTICE

5. Answers may vary. Primitive triples: 3, 4, 5; 7, 24, 25; 8, 15, 17; 5, 12, 13; Multiple of primitive triples: 6, 8, 10; 12, 16, 20; 14, 48, 50; 10, 24, 26; 9, 12, 15; 16, 30, 34; 20, 48, 52; 24, 45, 51

6. 50

7. There are two possibilities: $(-5, 1)$ and $(3, 1)$. Both create a 3-4-5 right triangle.

8. Counterexample: 3, 4, 5 is a Pythagorean triple, but adding 1 to each number yields 4, 5, 6, which is not a Pythagorean triple.

9. No; a Pythagorean triple must consist of whole numbers. Multiplying by 1.5 does not give a set of three whole numbers.

ADAPT

Debrief the Lesson Practice with students to gauge their understanding of Pythagorean triples. In particular, be sure students understand that Pythagorean triples must consist of three whole numbers. This means that 5.4, 7.2, and 9 do not form a Pythagorean triple even though $5.4^2 + 7.2^2 = 9^2$. If students have not yet mastered this content, assign additional problems from the Activity Practice.

My Notes

Check Your Understanding

3. How many Pythagorean triples can be created by multiplying the side lengths in a known triple by a constant? Explain your answer.

4. Do the numbers 65, 156, and 169 form a Pythagorean triple? Why or why not?

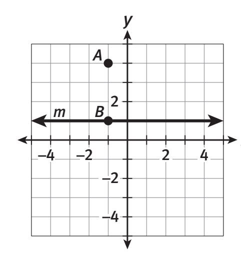
LESSON 24-2 PRACTICE

5. Below are sets of triangle side lengths. Sort the sets of lengths into two groups. Explain how you grouped the sets.

3, 4, 5	6, 8, 10	5, 12, 13	14, 48, 50
10, 24, 26	8, 15, 17	9, 12, 15	16, 30, 34
7, 24, 25	20, 48, 52	24, 45, 51	12, 16, 20

6. What number forms a Pythagorean triple with 14 and 48?

7. Point *C* is located on line *m*. What is the location of point *C* if the side lengths of $\triangle ABC$ form a Pythagorean triple? Is there more than one possibility? Explain.



8. Lisa says that if you start with a Pythagorean triple and add the same whole number to each number in the set, then the new set of numbers will also be a Pythagorean triple. Explain why Lisa is correct or provide a counterexample to show that she is not correct.

9. **Critique the reasoning of others.** Devon knows that 5, 12, 13 is a Pythagorean triple. He states that he can form a new Pythagorean triple by multiplying each of these values by 1.5. Is Devon correct? Justify your answer.

ACTIVITY 24 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 24-1

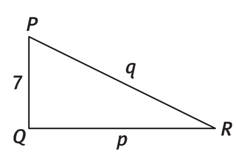
1. Is a triangle with sides measuring 9 feet, 12 feet, and 18 feet a right triangle? Justify your answer.
2. Determine whether $\frac{4}{5}$, $\frac{3}{5}$, and 1 can be the sides of a right triangle. Justify your answer.
3. The lengths of four straws are listed below. Which three of the straws can be placed together to form a right triangle? Why?

   5 cm

   6 cm

   12 cm

   13 cm
4. The lengths of the three sides of a right triangle are three consecutive even integers. What are they?
5. Which equation guarantees that $\triangle PQR$ is a right triangle?



A. $q^2 + 49 = p$
B. $q^2 - 7 = p^2$
C. $q^2 - 49 = p^2$
D. $q^2 + 7 = p$

Determine whether each statement is true or false. If the statement is false, explain why.

6. If a triangle has sides of length 8 cm, 10 cm, and 12 cm, then the triangle does not contain a right angle.
7. If you have sticks that are 15 in., 36 in., and 39 in. long, you can place the sticks together to form a triangle with three acute angles.
8. A triangle that has sides of length 7.5 cm, 10 cm, and 12.5 cm must be a right triangle.
9. The converse of the Pythagorean theorem says that in a right triangle the sum of the squares of the lengths of the legs equals the square of the hypotenuse.

Lesson 24-2

10. Is 9, 40, 41 a Pythagorean triple? Explain your reasoning.
11. The numbers 3, 4, 5 form a Pythagorean triple. Give four other Pythagorean triples that can be generated from this one.
12. Keiko said that the numbers 3.6, 4.8, and 6 form a Pythagorean triple since $6^2 = 3.6^2 + 4.8^2$. Do you agree or disagree? Explain.
13. Consider the following sets of whole numbers. Which sets form Pythagorean triples?

    I. 6, 8, 10

    II. 15, 36, 39

    III. 10, 12, 14

    IV. 16, 30, 34

A. I only
B. II and III
C. III and IV
D. I, II, and IV

ACTIVITY 24 Continued

ACTIVITY PRACTICE

1. No; $18^2 \neq 9^2 + 12^2$
2. Yes; $\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2 = 1$
3. 5 cm, 12 cm, 13 cm; $13^2 = 5^2 + 12^2$
4. 6, 8, 10
5. C
6. True; since $12^2 \neq 8^2 + 10^2$, the triangle is not a right triangle.
7. False; since $39^2 = 15^2 + 36^2$, the triangle is a right triangle.
8. True; $12.5^2 = 7.5^2 + 10^2$
9. False; this is the statement of the theorem, not its converse.
10. Yes; $41^2 = 40^2 + 9^2$
11. Answers may vary. 6, 8, 10; 9, 12, 15; 12, 16, 20; 15, 20, 25
12. Disagree; in a Pythagorean triple, the numbers must be whole numbers.
13. D

14. C
15. Disagree; multiplying each number in 3, 4, 5 by 8 gives another Pythagorean triple that includes 24 (24, 32, 40).
16. Answers may vary. 3, 4, 5
17. The numbers in a Pythagorean triple can be used to form the lengths of the sides of a right triangle.
18. sometimes
19. never
20. never
21. always
22. B
23. a. Answers may vary. Using $m = 3$ and $n = 2$, the formula generates the numbers 5, 12, and 13, which is a Pythagorean triple.
    b. No; this gives 0 for the value of a.

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.

14. Which whole number should be included in the set {8, 15} so that the three numbers form a Pythagorean triple?

A. 5 **B.** 12
C. 17 **D.** 19

15. Mario said the Pythagorean triple 7, 24, 25 is the only Pythagorean triple that includes the number 24. Do you agree or disagree? Justify your response.

16. Give an example of a Pythagorean triple that includes two prime numbers.

17. Explain the connection between Pythagorean triples and right triangles.

Determine whether each statement is always, sometimes, or never true.

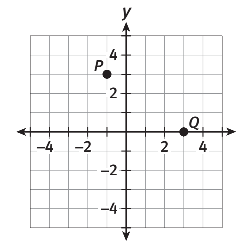
18. A Pythagorean triple includes an odd number.

19. Two of the numbers in a Pythagorean triple are equal.

20. A Pythagorean triple includes a number greater than 3 and less than 4.

21. The greatest number in a Pythagorean triple can be the length of the hypotenuse of a right triangle while the other two numbers can be the lengths of the legs.

22. The lengths of the sides of $\triangle PQR$ form a Pythagorean triple. Which of the following could be the coordinates of point R?



A. $(-3, -3)$ **B.** $(3, 3)$
C. $(3, -3)$ **D.** $(0, 3)$

MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

23. Euclid's formula is a formula for generating Pythagorean triples. To use the formula, choose two whole numbers, m and n, with $m > n$. Then calculate the following values.

$$a = m^2 - n^2$$
$$b = 2mn$$
$$c = m^2 + n^2$$

a. Choose values for m and n. Then generate the numbers a, b, and c according to the formula. Is the resulting set of numbers a Pythagorean triple?

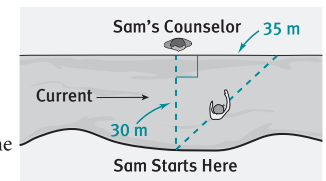
b. Does the formula work when $m = n$? Why or why not?

The Pythagorean Theorem
CAMP EUCLID

Embedded Assessment 4
Use after Activity 24

Sam is spending part of his summer vacation at Camp Euclid with some of his friends. On the first day of camp, they must pass an open-water swimming test to be allowed to use the canoes, kayaks, and personal watercraft. Sam and his friends must be able to swim across the river that they will be boating on.

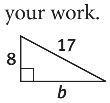
The river is 30 meters wide. On the day of the test, Sam begins on one bank and tries to swim directly across the river to the point on the opposite bank where his counselor is waiting. Because the river has a slight current, Sam ends up 35 meters downstream from his counselor.

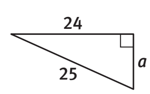


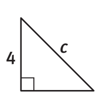
1. Copy and label the diagram for the problem situation.
2. How far did Sam actually swim? Justify your answer.
3. Sam's friend Alex started at the same spot but swam 50 meters. How far downstream was Alex from their counselor when he arrived at the opposite bank? Justify your answer.

In a lake fed by the river, a triangular area marked with buoys is roped off for swimming during free time at camp. The distances between each pair of buoys are 40 meters, 50 meters, and 60 meters.

4. Draw and label a diagram for the problem situation.
5. Is the swimming area a right triangle? Justify your answer.
6. Find the missing side length in each of the following triangles. Show all your work.

a.


b.


c.


7. Determine which of the following sets of triangle side lengths form right triangles. Justify each response.
   a. 9, 40, 41
   b. 20, 21, 31
   c. $\frac{6}{7}, \frac{8}{7}, \frac{10}{7}$
8. After the swimming test, Alex makes his way back to camp. On a coordinate plane, Alex is at the point $(-4, 3)$ and camp is at the point $(3, -1)$. What is the shortest distance Alex will have to travel to get back to camp? Assume each unit of the coordinate plane represents one kilometer.

Common Core State Standards for Embedded Assessment 4

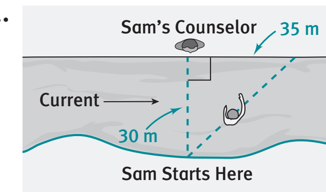
8.G.B.6	Explain a proof of the Pythagorean Theorem and its converse.
8.G.B.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
8.G.B.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

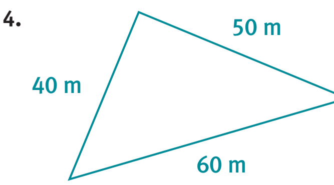
Embedded Assessment 4

Assessment Focus

- Solve problems using the Pythagorean Theorem
- Use the converse of the Pythagorean Theorem

Answer Key

1.


2. He swam 46.1 meters.
3. He was 40 meters from the counselor.
4.


5. No. $60^2 \neq 50^2 + 40^2$
6. a. $b = 15$
   b. $a = 7$
   c. $\sqrt{32} \approx 5.66$
7. a. Yes. 9, 40, 41 is a Pythagorean triple.
   b. No. $31^2 \neq 20^2 + 21^2$
   c. Yes. $\left(\frac{10}{7}\right)^2 = \left(\frac{6}{7}\right)^2 + \left(\frac{8}{7}\right)^2$
8. $\sqrt{65} \approx 8.06$ km

Teacher to Teacher

You may wish to read through the scoring guide with students and discuss the differences in the expectations at each level. Check that students understand the terms used.

Unpacking Embedded Assessment 5

Once students have completed this Embedded Assessment, turn to Embedded Assessment 5 and unpack it with students. Use a graphic organizer to help students understand the concepts they will need to know to be successful on Embedded Assessment 5.

Embedded Assessment 4
Use after Activity 24

The Pythagorean Theorem
CAMP EUCLID

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 2, 3, 5, 6a-c, 7a-c, 8)	• Using the Pythagorean Theorem to accurately find missing triangle side lengths and distance in the coordinate plane. • Using the converse of the Pythagorean Theorem to correctly determine if a triangle is a right triangle.	• Using the Pythagorean Theorem to find missing triangle side lengths and distance in the coordinate plane with few errors. • Using the converse of the Pythagorean Theorem to decide if a triangle is a right triangle.	• Difficulty in finding missing triangle side lengths and distance in the coordinate plane. • Difficulty determining if a triangle is a right triangle.	• Little or no understanding of using the Pythagorean Theorem. • Little or no understanding of using the converse of the Pythagorean Theorem.
Problem Solving (Items 2, 3, 5, 6, 7, 8)	• An appropriate and efficient strategy that results in a correct answer.	• A strategy that may include unnecessary steps but is correct.	• A strategy that results in some incorrect answers.	• No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 1, 4)	• Precisely modeling a problem situation with an accurate diagram.	• Drawing a reasonably accurate diagram to model a problem situation.	• Difficulty drawing a diagram to model a problem situation.	• Drawing an incorrect diagram to model a problem situation.
Reasoning and Communication (Items 2, 3, 5, 7)	• Correctly using the Pythagorean Theorem to justify answers to problems.	• Explaining an answer using the Pythagorean Theorem.	• Difficulty using the Pythagorean Theorem to justify answers.	• Little or no understanding of the Pythagorean Theorem.

Surface Area

Greenhouse Gardens

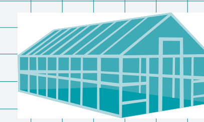
Lesson 25-1 Lateral and Surface Areas of Prisms

Learning Targets:

- Find the lateral and surface areas of rectangular prisms.
- Find the lateral and surface areas of triangular prisms.

SUGGESTED LEARNING STRATEGIES: Create Representations, Visualization, Think-Pair-Share

My Notes



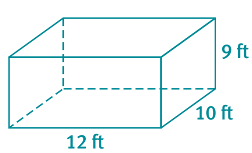
A greenhouse is a building used to grow plants. These buildings can vary widely in size and shape. By using a greenhouse, a gardener is able to grow a wider range of plants. The greenhouse shelters plants from weather and insects that can cause damage.

Marie and Ashton are planning to help build a greenhouse for their middle school. The local gardening club is donating funds and materials to get the greenhouse built.

When a diagram like the one above accompanies a verbal description, use the visual along with the scenario to activate prior knowledge. For example, identify geometric shapes you see in the greenhouse and review formulas for finding perimeter and area of those figures. Review with your group any background information that will be useful in applying these concepts as you solve the item below.

1. Marie looks at the first design for the greenhouse. The design is a rectangular prism with a length of 12 feet, a width of 10 feet, and a height of 9 feet. Sketch a model of the greenhouse.

   Sample sketch:



Marie and Ashton are asked to determine the cost of the glass that will be used to build the greenhouse. Glass will cover all of the walls of the greenhouse and the roof.

As you read Example A, clarify and make notes about any terms or descriptions you do not understand. Be sure to mark the text and label diagrams.

ACTIVITY 25

Guided

Activity Standards Focus

In this activity, students calculate lateral and surface areas of prisms and cylinders. Students gain experience working with surface areas in purely mathematical problems and in problems that arise from real-world situations.

Lesson 25-1

PLAN

Materials

- calculators
- shoebox

Pacing: 1 class period

Chunking the Lesson

#1 Example A #2 #3–5

Check Your Understanding

#8–9

Lesson Practice

TEACH

Bell-Ringer Activity

Show a shoebox with a cover to students. Ask students to draw a sketch of the box. Explain that the shoebox is an example of a rectangular prism. Encourage students to include dashed lines for the hidden sides/edges of the prism. Have students share their sketches and ask them to share any tricks or techniques they used to make the drawings look more realistic. This will serve as a good introduction to the figures students will be working with in this lesson.

1 Activating Prior Knowledge, Create Representations, Visualization, Think-Pair-Share, Debriefing

Students should recall what a rectangular prism is from earlier math classes. Check students' sketches to make sure they have created a reasonably accurate representation of the greenhouse.

Example A Activating Prior Knowledge, Create Representations, Visualization As you work through the example, be sure students understand that the surface area of the prism consists of six rectangular faces. Therefore, calculating the surface area involves finding the area of each of the six rectangles. The example shows how to do this in an organized way using the fact that pairs of opposite faces are congruent and have the same area.

Developing Math Language

This lesson introduces the terms *surface area* and *lateral area*. As needed, review the meanings of words that students may not have encountered in the past to help them understand them in the context of this scenario. Connect to students' prior knowledge of area by pointing out that the individual areas of the shapes that make up the solid figure are found first. Use visual and contextual support of a solid figure, such as a shoebox, to help explain the complex term.

Teacher to Teacher

Be sure students understand that calculating the surface area of a rectangular prism normally involves finding the total area of six faces. However, in this real-world context, the greenhouse does not have glass on the bottom face and so the surface area consists of the total area of the five faces. You may want to provide a net of a rectangular prism so that students can visualize the greenhouse and the areas that must be found.

My Notes

MATH TERMS

The **surface area** of a solid is the sum of the areas of all faces including the bases.

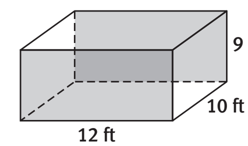
The **lateral area** of a solid is the sum of the areas of all faces excluding the bases. In a rectangular prism, you can assume the bases are the top and bottom faces, unless otherwise specified.

Example A

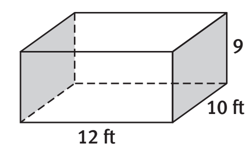
What is the surface area of the greenhouse that will be covered with glass? How would the lateral area differ from the surface area? Show all of your work.

Step 1: Identify the relevant faces. The ***surface area*** of a prism includes all faces. In this case, there is no glass on the bottom face, so find the area of the other five faces.

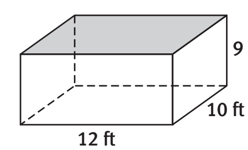
Step 2: Find the area of the front and back faces.
$2(12 \times 9) = 216 \text{ ft}^2$



Step 3: Find the area of the left and right faces.
$2(10 \times 9) = 180 \text{ ft}^2$



Step 4: Find the area of the top.
$1(12 \times 10) = 120 \text{ ft}^2$



Step 5: Add the areas.
$216 + 180 + 120 = 516 \text{ ft}^2$

Solution: 516 ft^2 will be covered with glass. The ***lateral area*** does not include the top or bottom faces, so the lateral area is $216 + 180 = 396 \text{ ft}^2$.

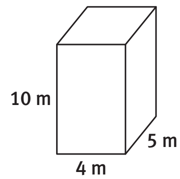
Lesson 25-1
Lateral and Surface Areas of Prisms

My Notes

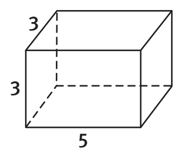
Try These A

Find the surface area and lateral area of the figures below. Show all work.

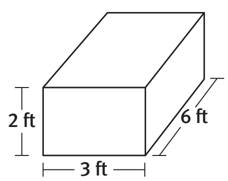
a.



b.



c.



2. The glass for the greenhouse costs $8 per square foot. What will be the cost of the glass if this design is used?

$516 \times 8 = \$4{,}128$

Try These A

a. $LA = 180$ in^2; $SA = 220$ in^2

b. $LA = 48$ square units; $SA = 78$ square units

c. $LA = 36$ ft^2; $SA = 72$ ft^2

2 Activating Prior Knowledge, Think-Pair-Share, Debriefing Have students share their work for this item. Check that students use correct units in their solution. In particular, the surface area of the greenhouse is expressed in square feet and the cost of the glass is given in dollars per square foot. This means the total cost of the glass for the greenhouse should be expressed in dollars.

3–5 Visualization, Identify a Subtask, Think-Pair-Share, Debriefing In Item 4, students may struggle to find the length of the third side of the triangular face of the prism. Pose the question, "What type of triangles form the bases of the prism?" (right triangles) Ask, "What relationship can you use to find the unknown side length?" (Pythagorean Theorem) Students will need to know the length of this side to calculate the lateral area and surface area in these items.

Teacher to Teacher

Have a brief class discussion about the relationship between the surface area and the lateral area of a prism. This will help students realize that the surface area must always be greater than the lateral area. This fact can be used as a simple way to assess the reasonableness of answers. In particular, students' answer to Item 5 should be greater than their answer to Item 4.

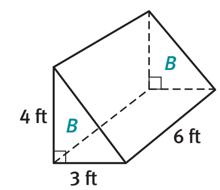
ELL Support

Some students may need review of the Pythagorean Theorem for right triangles.

Assign the mini-lesson below to give students practice solving for the length of an unknown side of a right triangle.

My Notes

3. In the right triangular prism below, mark the bases with a *B*. What two-dimensional shapes make up the lateral area of the figure?
The lateral area is made up of three rectangles.

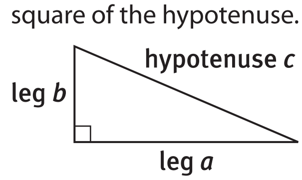


4. Ashton is building containers to hold plant food in the greenhouse. The design for these containers is shown above. Ashton will use plywood to cover the lateral area of the right triangular prism. What is the lateral area of the container? Show your work.
$3 \times 6 = 18$ ft^2
$4 \times 6 = 24$ ft^2
$5 \times 6 = 30$ ft^2
$LA = 18 + 24 + 30 = 72$ ft^2

5. Attend to precision. Marie is working with Ashton on building the containers. She suggests that Ashton cover the bases with plywood as well. Ashton realizes he needs to find the total surface area of the right triangular prism to determine how much plywood he needs. Calculate the surface area of the container.
$B = \frac{1}{2}(3)(4) = 6$ ft^2
$SA = 72 + 2(6) = 84$ ft^2

MINI-LESSON: The Pythagorean Theorem

For any right triangle, the sum of the squares of the legs of the triangle is equal to the square of the hypotenuse.



$a^2 + b^2 = c^2$

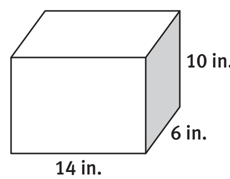
Find the length of the missing side. Round to the nearest tenth.

1. $a = 2, b = 9$ $\quad$ **$c = 9.2$**
2. $c = 10, b = 3$ $\quad$ **$a = 9.5$**

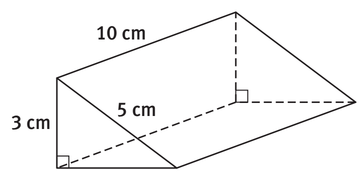
My Notes

Check Your Understanding

6. Find the lateral area and surface area of the rectangular prism.



7. Find the lateral area and surface area of the right triangular prism.



8. a. Sal is painting a bedroom with the dimensions of 12 feet long by 10 feet wide by 7 feet high. If he only paints the four walls of the room, how much area will he need to paint?
308 ft^2

b. Sal used this formula for indicating how much wall area to paint in a room. In this formula, l is the length of the room, w is the width, and h is the height, or distance from the floor to the ceiling.

$$L = 2 \cdot l \cdot h + 2 \cdot w \cdot h.$$

Explain how this formula gives the lateral area of the room.

c. Sal's sister Simone claims that the formula $L = Ph$, where P is the perimeter of the room and h is the height of the room, also works. Is Simone's claim correct? Explain.

d. If Sal decides to paint the total surface area of the room, including the floor and ceiling, what will he need to add to his formula? Explain. Then write a formula for total surface area using l for length, w for width, and h for height.

9. Make sense of problems. How many gallons of paint will Marie and Ashton need to paint five of the right triangular prisms that will hold the plant food from Item 5 if one gallon of paint covers about 200 square feet? Assume they paint all sides of the containers. Explain your answer.
They need to paint a total of $5(84) = 420 \text{ ft}^2$, so they need 3 gallons of paint.

Check Your Understanding:

Use these items as a formative assessment to check that students understand the difference between lateral area and surface area and that they can calculate both of these measures for rectangular prisms and triangular prisms. Debriefing students' methods will help struggling students internalize the steps for performing these calculations.

Answers

6. $LA = 400 \text{ in}^2$; $SA = 568 \text{ in}^2$
7. $LA = 120 \text{ cm}^2$; $SA = 132 \text{ cm}^2$

8–9 Visualization, Identify a Subtask, Think-Pair-Share, Debriefing, Self Revision/Peer Revision These items require students to apply what they know about lateral area and surface area to real-world problems. Suggest that students sketch the figures and label sides with known measurements. Have them label unknown sides with variables.

Students critique formulas for wall area in Items 8b and 8c. Encourage them to use proper math terms in explaining their formulas. For Item 8c, have students write a formula for the perimeter and substitute into Simone's formula. Then have them compare their work to the formula in Item 8a. Help students find the terms needed for Sal's formula in Item 8d by having them draw and label a floor and ceiling. Then they can find the formulas for the area of those two parts of the room. Students will then add these expressions to the formula for wall area to find total surface area.

8b. Answers will vary. There are four walls that make up the lateral surface area. One wall and the wall opposite have an area of $l \cdot h$, so these walls have a combined area of $2 \cdot l \cdot h$. The remaining two walls both have areas of $w \cdot h$, so these walls combined have an area of $2 \cdot w \cdot h$. All four walls combined have an area of $2 \cdot l \cdot h + 2 \cdot w \cdot h$.

8c. Answers will vary. Simone is correct. The perimeter can be written as $P = 2 \cdot l + 2 \cdot w$. When multiplied by the height, $Ph = (2 \cdot l + 2 \cdot w) \cdot h = 2 \cdot l \cdot h + 2 \cdot w \cdot h$. So Simone's formula is equivalent to Sal's formula.

8d. Answers will vary. Sal will need to add the area of the floor and ceiling, both of which have an area of $l \cdot w$. The formula for the total surface area is $S = 2 \cdot l \cdot h + 2 \cdot w \cdot h + 2 \cdot l \cdot w$.

ASSESS

Students' answers to lesson practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 25-1 PRACTICE

10. $LA = 324\text{ cm}^2$; $SA = 486\text{ cm}^2$

11. $LA = 400\text{ ft}^2$; $SA = 520\text{ ft}^2$

12. $LA = 70\text{ cm}^2$; $SA = 94\text{ cm}^2$

13. $LA = 256\text{ in}^2$; $SA = 320\text{ in}^2$

14. Box A has a surface area of 342 in^2. This is greater than the surface area of Box B (280 in^2), so Box A requires more cardboard to manufacture.

15. The 2-by-2-by-2 prism has the least surface area (24 square units); the 1-by-1-by-8 prism has the greatest surface area (34 square units).

ADAPT

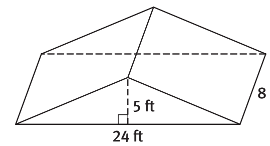
Check students' work to ensure that they can calculate lateral areas and surface areas of prisms. If students need further work with the concept, make some nets of rectangular and triangular prisms on grid paper to help students visualize and compute lateral area and surface area and determine the difference between them. If students need additional experience with this skill, assign problems from the Activity Practice.

My Notes

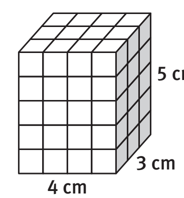
LESSON 25-1 PRACTICE

10. A cube is a rectangular prism with square faces. Suppose a cube has edges 9 cm long. What is its lateral area? What is its surface area?

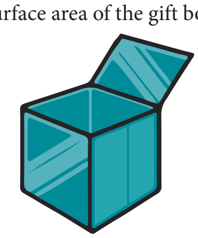
11. Find the lateral area and surface area of the triangular prism.



12. The prism shown here is made from centimeter cubes. Find the lateral and surface area of the prism.



13. A gift box is a cube that measures 8 inches by 8 inches on all sides. What is the lateral and surface area of the gift box without the lid?

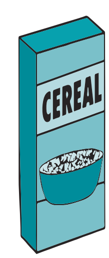


14. **Construct viable arguments.** Which cereal box requires more cardboard to manufacture? Justify your answer.

A.

depth = 3 in.
width = 9 in.
height = 12 in.

B.



depth = 2 in.
width = 7 in.
height = 14 in.

15. Use 8 cubes to create as many prisms as you can. Find and describe the prism with the least surface area and the greatest surface area.

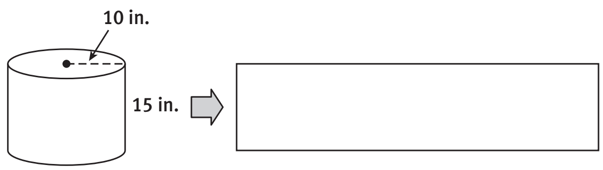
Lesson 25-2
Lateral and Surface Areas of Cylinders

Learning Targets:
- Find the lateral area of cylinders.
- Find the surface area of cylinders.

SUGGESTED LEARNING STRATEGIES: Create Representations, Visualization, Think-Pair-Share

1. Marie looks at sketches for the containers that will hold individual plants. The plant containers are in the shape of a cylinder. Marie wants to wrap decorative paper around the curved surfaces of the containers. The circular bases on the top and bottom of the containers will not be covered with the paper. What part of each cylinder will be covered?
The lateral surface of the cylinder will be covered.

2. **Make use of structure.** The design for the cylindrical plant containers is shown on the left below. The rectangle shows a strip of paper that perfectly fits the lateral surface of the cylinder, without gaps or overlap. Explain how to find the dimensions of this rectangle.



The height of the rectangle is the height of the cylinder (15 in.). The length of the rectangle is the circumference of the base of the cylinder ($2\pi r$ or 20π in.).

3. What does your answer to Item 2 tell you about how to find the lateral area of a cylinder?
The lateral area is the area of the rectangle, that is, the height of the cylinder times the circumference of the base.

My Notes

ACTIVITY 25 Continued

Lesson 25-2

PLAN

Materials
- calculators
- timer

Pacing: 1 class period

Chunking the Lesson
#1–3 Example A #4–5
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity
Set a timer for five minutes. Direct students to do a Quickwrite in which they list everything they know about cylinders. Encourage students to include a sketch in which they label the important parts of a cylinder. Debrief students' work with the class. This will serve as a good review of the essential vocabulary related to cylinders.

1–3 Create Representations, Visualization, Use Manipulatives, Think-Pair-Share It may be helpful to illustrate these items using the cardboard cylinder from a roll of paper towels. Have students discuss what part of the cylinder will be covered with decorative paper. Ask students to describe how the cylinder is related to the rectangle. Cut the cardboard cylinder down its length and flatten it. Show students that the circumference of the base is the length of the rectangle when the cardboard cylinder is flattened. The height of the cylinder is the height of the rectangle when the cardboard cylinder is flattened.

Example A Create Representations, Visualization, Think-Pair-Share, Debriefing As you discuss this example with students, ask them to compare finding the lateral area with finding the surface area of a prism. This will help students see that the lateral area is the area of the solid without the bases and the surface area is the lateral area plus the area of the bases.

Teacher to Teacher

As you discuss the example, remind students that it is best to round only as a last step. This results in the most accurate answer. Also, encourage students to use the π key on their calculators rather than 3.14 or other approximations.

Try These A

a. $LA = 600\pi \approx 1885.0$ cm^2 ;
$SA = 1050\pi \approx 3298.7$ cm^2

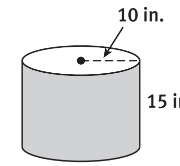
b. $LA = 168\pi$ in.$^2 \approx 527.8$ in.^{2};
$SA = 216\pi$ in.$^2 \approx 678.6$ in.^{2}

My Notes

Example A

Calculate the surface area of the plant container from Item 2.

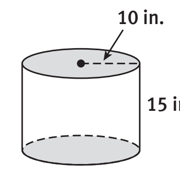
Step 1: Calculate the lateral area.



The lateral area is the area of the rectangle that covers the curved surface of the cylinder.

$LA =$ (height of cylinder) $\times$ (circumference of base)
$= (15)(2\pi \cdot 10) = 300\pi$

Step 2: Calculate the area of the bases.



Each base is a circle with area πr^2.
The area of each base is $\pi(10)^2 = 100\pi$.
So the total area of the two bases is $2 \cdot 100\pi = 200\pi$.

Step 3: Add the lateral area and the area of the bases.

$300\pi + 200\pi = 500\pi \approx 1570.8$ in.^{2}

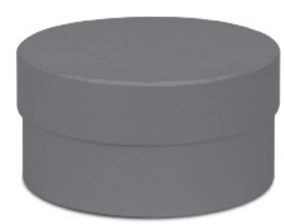
Solution: The surface area is 500π in.^{2} or approximately 1,570.8 in.^{2}.

Try These A

Find the lateral area and surface area of the objects below. Give your answers in terms of π and rounded to the nearest tenth.

a. a cylindrical hat box with diameter 30 cm and height 20 cm

b. a six-pack of juice cans where each can has a radius of 2 inches and a height of 7 inches

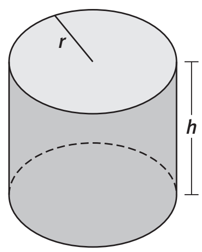


Lesson 25-2
Lateral and Surface Areas of Cylinders

My Notes

4. What are the lateral area and surface area of the cylinder below using variables r and h?

$LA = 2\pi rh$
$SA = 2\pi rh + 2\pi r^2$



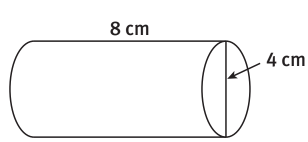
5. In the above figure, suppose $r = 5$ ft and $h = 15$ ft. What are the lateral area and surface area of the cylinder in this case? Show all work and leave your answers in terms of π.

$$
\begin{aligned}
LA &= 2\pi(5)(15) \\
&= 2\pi(75) \\
&= 150\pi \text{ ft}^2
\end{aligned}
$$

$$
\begin{aligned}
SA &= 150\pi + 2\pi(5)^2 \\
&= 150\pi + 2\pi(25) \\
&= 150\pi + 50\pi \\
&= 200\pi \text{ ft}^2
\end{aligned}
$$

Check Your Understanding

6. Find the lateral area and surface area of the cylinder. Leave your answers in terms of π.



7. Is it ever possible for the lateral area of a cylinder to equal the surface area of the cylinder? Justify your response.

ACTIVITY 25 Continued

4–5 Create Representations, Visualization, Think-Pair-Share, Debriefing These items provide a more abstract approach to the surface area of a cylinder. Item 4 asks students to use what they have learned so far to write a formula for the surface area of a cylinder. Item 5 gives students a chance to apply their formula. You might ask students how using the formula is similar to and different from the solution process that was shown in the example.

Check Your Understanding

These items serve as a formative assessment of students' understanding of the lateral area and surface area of a cylinder. Debriefing students' responses and their reasoning will also give you a sense of whether students can use the related terminology appropriately.

Answers

6. $LA = 32\pi$ cm^2 ; $SA = 40\pi$ cm^2

7. No; the surface area is equal to the lateral area plus the areas of the bases and the area of each base must be greater than 0.

ASSESS

Students' answers to lesson practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 25-2 PRACTICE

8. 10π in.^{2}
9. 18π in.^{2}
10. $\frac{\pi}{4}$ in.^{2}
11. $\frac{3\pi}{8}$ in.^{2}
12. 527.8 in.^{2}

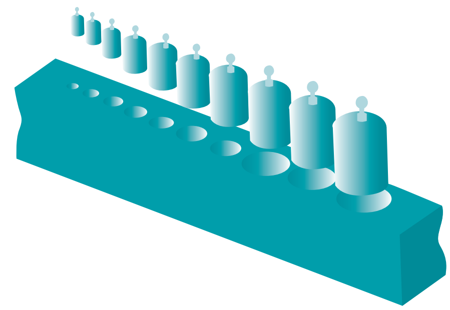
ADAPT

Check students' work to ensure that they can calculate lateral areas and surface areas of cylinders. Provide cylinder models such as soup or tuna cans with removable labels to help students visualize lateral and surface area. If students need additional experience with this skill, assign problems from the Activity Practice.

My Notes

LESSON 25-2 PRACTICE

This cylinder activity is used to promote fine-motor skills and spatial learning in young children. Use the figure for Items 8–11 and leave your answers in terms of π.



8. Calculate the lateral area of the largest peg to the far right of the diagram if its height is 2.5 inches and radius is 2 inches.

9. What is the surface area of the largest peg?

10. What is the lateral area of the smallest peg to the far left of the diagram if its height is $\frac{1}{2}$ inch and radius is $\frac{1}{4}$ inch?

11. What is the surface area of the smallest peg?

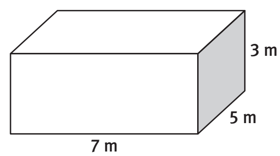
12. **Model with mathematics.** A water bottle has a label that is 3.5 inches high. If the bottle has a radius of 2 inches, how much paper would be needed to put labels on one dozen water bottles? Round to the nearest tenth.

ACTIVITY 25 PRACTICE

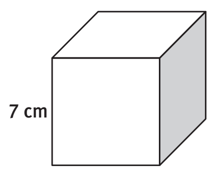
Write your answers on notebook paper.
Show your work.

Lesson 25-1

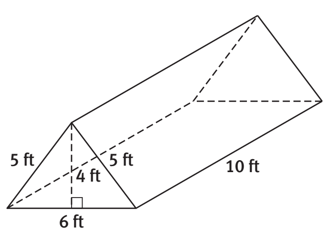
1. Find the surface area of the rectangular prism shown below.



2. Find the surface area of the cube shown below.



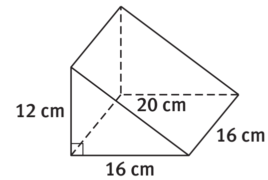
3. The dimensions of a nylon tent are shown in the figure.



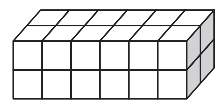
a. How much nylon is needed to make the sides and floor of the tent?
b. How much nylon is needed to make the triangular flaps at the front and back of the tent?
c. What is the surface area of the triangular prism? How is this related to your responses to parts a and b?

4. A gift box is 6 inches long, 3 inches wide, and 3 inches tall.
a. How much paper is needed to wrap the box? Assume the box is wrapped with the minimum amount of paper and no overlap.
b. How much wrapping paper should you buy to wrap the box if you assume you will need 15% extra for waste and overlap?

5. Find the lateral area and surface area of the triangular prism.



The figure below is made from one-inch cubes. Use the figure for Items 6 and 7.

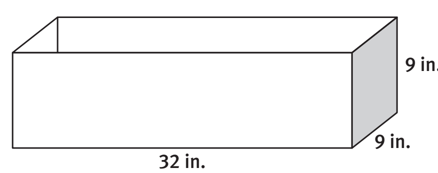


6. What are the dimensions of the figure?

7. What is the surface area of the figure?

8. A cube has a surface area of 96 m^2. What is the length of each edge of the cube?
A. 2 m **B.** 4 m
C. 6 m **D.** 8 m

9. A window box for flowers is a rectangular prism with an open top, as shown. Tyrell wants to coat the inside and outside of the box with a special varnish that will protect it from the effects of water, cold, and harsh weather. The varnish comes in cans that can cover 500 square inches. How many cans of the varnish should Tyrell buy? Explain your answer.



10. Which is the best estimate of the lateral area of a cube with edges that are 2.1 inches long?
A. 9 in.^{2} **B.** 16 in.^{2}
C. 25 in.^{2} **D.** 36 in.^{2}

ACTIVITY 25 Continued

ACTIVITY PRACTICE

1. 142 m^2
2. 294 cm^2
3. a. 150 ft^2
b. 24 ft^2
c. 174 ft^2; this is the total amount of nylon needed to make the tent.
4. a. 90 in.^{2}
b. 103.5 in.^{2}
5. $LA = 768$ cm^2; $SA = 960$ cm^2
6. 6 inches long, 2 inches wide, 2 inches high
7. 56 in.^{2}
8. B
9. 5 cans; the area to be painted is two times the outside area or $2 \times 1026 = 2052$ in.^{2}
10. C

ACTIVITY 25 Continued

ACTIVITY PRACTICE

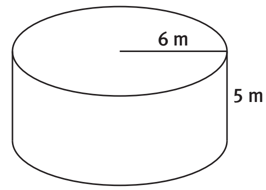
11. $LA = 60\pi$ m^2; $SA = 132\pi$ m^2
12. 377.0 cm^2
13. C
14. 113.1 ft^2
15. A
16. D
17. **a.** 80π cm^2
b. 105π cm^2
c. Disagree; doubling the radius changes the total area to 260 cm^2.
18. 3.6 ft^2
19. **a.** 4 cm^2; 8 cm^2; 16 cm^2; 32 cm^2
b. When the radius is doubled, the lateral area also doubles.

ADDITIONAL PRACTICE

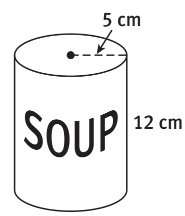
If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.

Lesson 25-2

11. Find the lateral area and surface area of the cylinder. Leave your answers in terms of π.



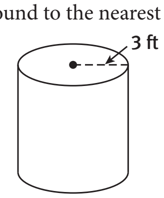
12. What is the area of the label on the soup can shown below? Round to the nearest tenth.



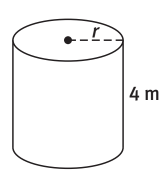
13. An orange juice can has a diameter of 4 inches and a height of 7 inches. The curved surface of the can is painted orange. How much paint is needed?

A. 14π in.^{2} **B.** 16π in.^{2}
C. 28π in.^{2} **D.** 36π in.^{2}

14. The height of the cylinder shown below is twice the radius. What is the lateral area of the cylinder? Round to the nearest tenth.

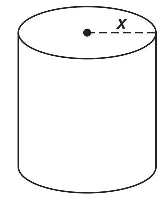


15. The lateral area of the cylinder shown below is 12π m^2. What is the radius of the cylinder?



A. 1.5 m **B.** 3 m
C. 6 m **D.** 8 m

16. Which expression best represents the surface area of the cylinder shown below?

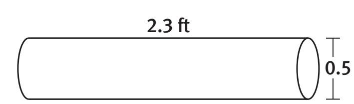


A. $2\pi x + 2\pi x^2$ **B.** $2\pi x + 4\pi x^2$
C. $4\pi x + 4\pi x^2$ **D.** $4\pi x + 2\pi x^2$

17. Mei is designing a cylindrical container for her ceramics class. The container will be open on top. She is considering a container with a radius of 5 cm and a height of 8 cm.

a. Find the lateral area of the container. Leave your answer in terms of π.
b. What is the total area of the container? Leave your answer in terms of π.
c. Mei's friend Victor states that if she doubles the radius of the container she will double the total area of the container. Do you agree or disagree? Justify your response.

18. A pipe is made from a thin sheet of copper. The dimensions of the pipe are shown below. What is the amount of copper needed to make the pipe? Round to the nearest tenth.



MATHEMATICAL PRACTICES

Look For and Express Regularity in Repeated Reasoning

19. Consider a cylinder with a height of 1 cm.

a. Find the lateral area of the cylinder if the radius is 2 cm, 4 cm, 8 cm, and 16 cm. Leave your answers in terms of π.
b. Use your results to make a conjecture about what happens to the lateral area of a cylinder when the radius is doubled.

Volumes of Solids

ACTIVITY 26

Castles in the Sand

Lesson 26-1 Volumes of Prisms and Pyramids

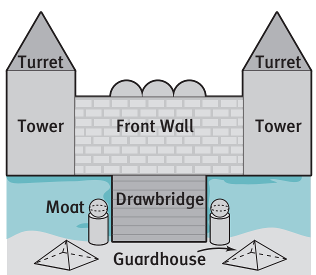
Learning Targets:

- Apply the formula for the volume of a prism.
- Apply the formula for the volume of a pyramid.

SUGGESTED LEARNING STRATEGIES: Create Representations, Visualization, Think-Pair-Share, Group Presentation

The eighth-grade class at LWH Middle School in Montana hosted a spring festival to raise money for their end-of-year trip to the beach. They decided to sponsor a sand castle–building contest as part of the festivities. Because the sand for the sand castles had to be trucked in to the school, students who wanted to participate in the castle-building contest were required to submit a proposal to Archie Medes, the geometry teacher. The proposals had to contain a sketch of the castle the student or group of students wanted to build, a list of the solids used and their dimensions, and the volume of sand required to build it.

Shayla wanted to enter the contest. She decided to research castles to help brainstorm ideas for her proposal. One of the castles she looked at was Fantasy Castle in a nearby theme park.



1. What solids could Shayla use to build a sand replica of Fantasy Castle?
   Sample answers: prism, pyramid, cylinder, cone, sphere

Shayla's friend Shelly built last year's winning castle out of prisms, pyramids, cylinders, cones, and spheres. To help Shayla prepare for the contest, Shelly showed Shayla her plans from last year and explained how she determined the amount of sand she would need to build her winning castle.

My Notes

Common Core State Standards for Activity 26

8.G.C.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

ACTIVITY 26

Guided

Activity Standards Focus

In this activity, students apply the formula for the volume of a prism, pyramid, cylinder, cone, and sphere in a variety of mathematical and real-world situations. Students also apply what they have learned to find the volume of composite solids.

Lesson 26-1

PLAN

Materials

- calculators
- pictures/models of rectangular prisms and pyramids
- timer

Pacing: 1 class period

Chunking the Lesson

#1 Examples A–B #2–3
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity

Set a timer for 5 minutes. Show photographs or models of prisms and pyramids. Ask students to compare and contrast the figures by focusing on the attributes such as names of the face shapes, and the number of faces and vertices. This provides visual and contextual support to enhance and confirm students' knowledge of these figures. This activity also serves as a natural segue into the content of this lesson, which focuses on finding volumes of solids, including prisms and pyramids.

1 Activating Prior Knowledge, Shared Reading, Visualization, Think-Pair-Share Students should share reading the introduction to this item and discuss various sand castles that they may have seen. They will see a variety of solid figures when looking at the picture of this castle. Discussing the solid figures will help all students to identify the variety of solid figures that exist in this castle and recall the properties of the figures.

Teacher to Teacher

Students may not be familiar with castles and the names for the parts of a castle. Displaying pictures of castles and discussing the parts of a castle helps to develop students' backgrounds to comprehend the special, challenging words related to castles.

Examples A–B Shared Reading, Create Representations, Visualization, Think-Pair-Share As you discuss the examples with students, ask them to look for commonalities in the two solutions. Be sure students realize that the volume formulas for these two solids are the same, with the exception of the factor of $\frac{1}{3}$ in the formula for the volume of a pyramid. Also be sure students pay careful attention to units. They should understand that the volume of a solid is always expressed in cubic units, such as cubic centimeters or cubic inches.

My Notes

MATH TIP

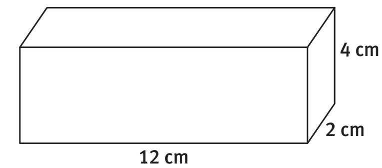
The formula for the volume V of a prism is $V = Bh$, where B is the area of the base and h is the height. For a rectangular prism with length ℓ, width w, and height h, the formula may be written as $V = \ell wh$.

MATH TIP

The formula for the volume V of a pyramid with base area B and height h is $V = \frac{1}{3}Bh$.

Example A

Find the volume of the rectangular prism shown below.



Step 1: Write the volume formula, $V = \ell wh$. Identify the values of the variables.

$$\ell = 12 \text{ cm},\ w = 2 \text{ cm, and } h = 4 \text{ cm}$$

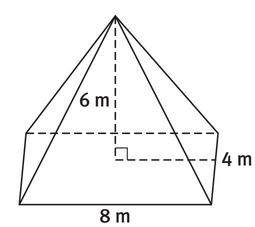
Step 2: Substitute the dimensions into the formula.

$$\begin{aligned} V &= \ell wh \\ &= 12 \bullet 2 \bullet 4 = 96 \end{aligned}$$

Solution: $V = 96 \text{ cm}^3$

Example B

Find the volume of the rectangular pyramid shown below.



Step 1: Write the volume formula, $V = \frac{1}{3}Bh$. Identify the known values of the variables.

$$h = 6 \text{ m}$$

Step 2: Calculate the area of the base.

$$B = 8 \times 4 = 32 \text{ m}^2$$

Step 3: Substitute the dimensions into the formula.

$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}(32)(6) = 64 \end{aligned}$$

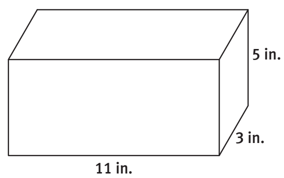
Solution: $V = 64 \text{ m}^3$

My Notes

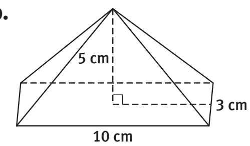
Try These A–B

Find the volume of each solid.

a.

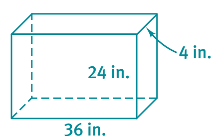


b.



2. Shayla begins to plan the front wall of the castle.

a. Draw and label a sketch of the prism used for the front wall of the castle if the wall is 36 inches long, 24 inches high, and 4 inches wide.

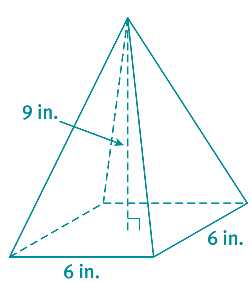


b. Use the volume formula for a prism to find the number of cubic inches of sand needed to build the front wall.

$V = 3{,}456$ in.^{3}

3. Make sense of problems. Next, Shayla plans the guardhouses in front of the moat.

a. Draw and label a sketch of one of the pyramids used for the guardhouses at the entrance to the drawbridge given that each pyramid has a height of 9 inches and given that the base of each pyramid is a square with side lengths of 6 inches.



b. Use the volume formula for a pyramid to find the number of cubic inches of sand needed to build both guardhouses.

216 in.^{3}

Try These A–B

a. 165 in.^{3}
b. 50 cm^3

2–3 Shared Reading, Create Representations, Visualization, Think-Pair-Share, Group Presentation Remind students to use the formula sheet located in the back of the book. Students should share their sketches, work, and answers with the group. As the activity develops, continue to work with students on labeling their sketches accurately by placing dimensions where they belong in the sketch. Group presentations will help this process, allowing students to view the work of others and edit their own work.

ELL Support

Some students may have a difficult time with the sketching involved in this activity. Encourage students to think about the two-dimensional shapes involved in creating each sketch and ask them to consider what would be a good starting place in making each sketch. It can be helpful to have some diagrams and models of the various solids in the classroom so that students have a reference for their sketches.

Check Your Understanding

Use these items as a formative assessment to check that students can sketch a pyramid or prism based on a verbal description and then calculate its volume. Debriefing students' responses with the class will be helpful to students who may need extra support to master these skills.

ASSESS

Students' answers to lesson practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 26-1 PRACTICE

6. Use square millimeters (mm^2) for the surface area and cubic millimeters (mm^3) for the volume.
7. 70 cm^3
8. 10 in.
9. 120 cm^3
10. **a.** 84 $in.^3$
b. 84 blocks; pack a 7-by-4 layer on the bottom of the box, then another 7-by-4 layer on top of this, and finally a 7-by-4 layer on the top.
c. The results are the same. This makes sense since the volume of the rectangular prism is the number of unit cubes (cubes of volume 1 $in.^3$) that it can hold.

ADAPT

Check students' work to ensure that they can calculate volumes of prisms and pyramids in a variety of mathematical and real-world settings. If students need additional practice, assign problems from the Activity Practice. Also, students will have more opportunities to work with these solids in Lesson 26-3 when they calculate volumes of composite solids.

My Notes

Check Your Understanding

4. Draw a square pyramid with a height of 8 centimeters and base side lengths of 6 centimeters. Find the volume.

5. Draw a cube with side lengths of 4 inches. Find the volume.

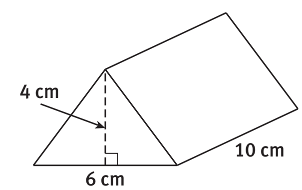
LESSON 26-1 PRACTICE

6. Julian measures the edges of a box in millimeters. What units should he use for the surface area of the box? What units should he use for the volume of the box?

7. Find the volume of a triangular prism with a base area of 14 square centimeters and a height of 5 centimeters.

8. A triangular pyramid has a volume of 20 $in.^3$. The base area of the pyramid is 6 $in.^2$. What is the height of the pyramid?

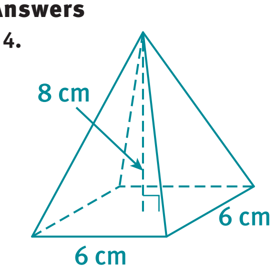
9. Find the volume of the solid shown below.



10. **Reason quantitatively.** A toy manufacturer makes alphabet blocks in the shape of cubes with a side length of 1 inch.
a. The manufacturer plans to pack the blocks in a box that is a rectangular prism. The box is 7 inches long, 4 inches wide, and 3 inches tall. What is the volume of the box?
b. Suppose the manufacturer packs the blocks efficiently, so that as many blocks fit in the box as possible. How many blocks can fit? Describe how they would be packed.
c. Describe the connection between your answers to parts a and b.

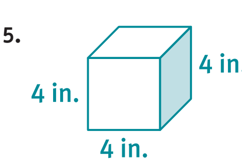
Answers

4.



volume: 96 cm^3

5.



volume: 64 $in.^2$

My Notes

Learning Targets:

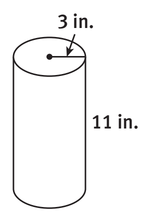
- Apply the formula for the volume of a cone.
- Apply the formula for the volume of a cylinder.
- Apply the formula for the volume of a sphere.

SUGGESTED LEARNING STRATEGIES: Create Representations, Think-Pair-Share, Group Presentation, Quickwrite, Visualization

Shayla's friend Shelly continues to help Shayla prepare for the contest by showing her how to calculate the amount of sand needed to build various solids.

Example A

Find the volume of the cylinder.



Step 1: Write the volume formula, $V = Bh$. Identify the known values of the variables.

$$h = 11 \text{ in.}$$

Step 2: Calculate the area of the base.

$$B = \pi r^2 = \pi(3)^2 = 9\pi \text{ in.}^2$$

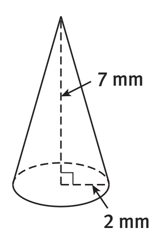
Step 3: Substitute the dimensions into the formula.

$$\begin{aligned} V &= Bh \\ &= 9\pi(11) \\ &= 99\pi \end{aligned}$$

Solution: $V = 99\pi \text{ in.}^3 \approx 311.02 \text{ in.}^3$

Example B

Find the volume of the cone.



Step 1: Write the volume formula, $V = \frac{1}{3}Bh$. Identify the known values of the variables.

$$h = 7 \text{ mm}$$

Step 2: Calculate the area of the base.

$$B = \pi r^2 = \pi(2)^2 = 4\pi \text{ mm}^2$$

Step 3: Substitute the dimensions into the formula.

$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}(4\pi)(7) \\ &= \frac{28\pi}{3} \end{aligned}$$

Solution: $V = \frac{28\pi}{3} \text{ mm}^3 \approx 29.32 \text{ mm}^3$

MATH TIP

The formula for the volume V of a cylinder is $V = Bh$, where B is the area of the base and h is the height. Since $B = \pi r^2$, the formula may be written as $V = \pi r^2 h$.

MATH TIP

The formula for the volume V of a cone is $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height. Since $B = \pi r^2$, the formula may be written as $V = \frac{1}{3}\pi r^2 h$.

Compare the volume formulas for a cylinder and a cone. How are they the same? How are they different?

Lesson 26-2

PLAN

Materials

- calculators

Pacing: 1–2 class periods

Chunking the Lesson

Examples A-B Example C
#1–3 #4–5
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity

Give students 3 or 4 minutes to compare and contrast cylinders and cones. Encourage students to list similarities and differences. Then have volunteers share their responses. This will help students come to understand that cones are related to cylinders in much the same way that pyramids are related to prisms.

Example A–B Activating Prior Knowledge, Shared Reading, Visualization, Think-Pair-Share

Discuss these examples with the class, stressing the common steps in the solution processes. Take a few moments to discuss the relevant formulas with the class and be sure students realize that the formula for the volume of a cone is the same as the formula for the volume of a cylinder except for the factor of $\frac{1}{3}$ in the formula for the volume of a cone.

TEACHER to TEACHER

As with all surface area and volume problems, remind students that it is best to round only as a final step and to use the π key on their calculator rather than an approximation such as 3.14.

MINI-LESSON: Prisms and Pyramids; Cylinders and Cones

In this mini-lesson, students work with a prism and pyramid with congruent bases and heights, and also with a cylinder and cone with congruent bases and heights. In both cases, students fill the smaller solid with rice or a similar item and then see how many of these it takes to fill the larger solid.

See SpringBoard's eBook Teacher Resources for a student page for this mini-lesson.

Example C Activating Prior Knowledge, Shared Reading, Visualization, Think-Pair-Share
Spheres may be somewhat less familiar to students than the solids they have worked with up to now. Point out that the formula for the volume of a sphere involves only one variable–the radius of the sphere.

Try These A-B-C
a. 288π m$^3 \approx 904.8$ m^3
b. 224π cm$^3 \approx 703.7$ cm^3
c. 12π in.$^3 \approx 37.7$ in.^{3}

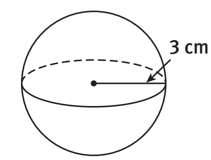
My Notes

MATH TIP

The formula for the volume V of a sphere is $V = \frac{4}{3}\pi r^3$

Example C

Find the volume of the sphere shown below.



Step 1: Write the volume formula, $V = \frac{4}{3}\pi r^3$.
Identify the values of the variables.
$r = 3$ cm

Step 2: Substitute the value of r into the formula.

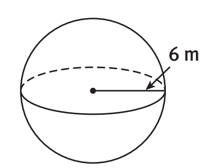
$$V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi(3)^3$$
$$= 36\pi$$

Solution: $V = 36\pi$ cm$^3 \approx 113.10$ cm^3

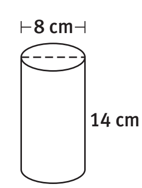
Try These A–B–C

Find the volume of each solid. Leave your answers in terms of π and round to the nearest tenth.

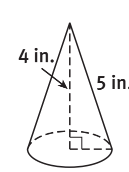
a.



b.



c.

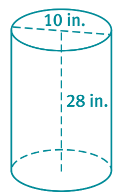


Lesson 26-2
Volumes of Cylinders, Cones, and Spheres

My Notes

1. Shayla begins work on the cylindrical towers on either side of the front wall.

a. Draw and label a sketch of one of the cylinders used to create the two towers if the diameter of each cylinder is 10 inches and the height of each cylinder is 28 inches.

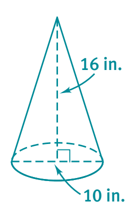


b. Use the volume formula for a cylinder to find the number of cubic inches of sand needed to build the two congruent cylindrical towers.
$\approx$ 4,398.2 in.^{3}

2. Shelly tells Shayla that she uses the formula $V = Bh$ to find the volume of both cylinders and prisms. Why does this work?
In both cases, the volume is the product of the area of the base times the height. In the case of a cylinder, $B = \pi r^2$.

3. Now Shayla works on the turrets.

a. Draw and label a sketch of one of the cones used to create the turrets if the diameter of the base of each cone is 10 inches and the height of each cone is 16 inches.



b. Attend to precision. Use the volume formula for a cone to find how many cubic inches of sand are needed to build the two congruent conical turrets. How many cubic feet are needed?
$\approx$ 837.8 in.^{3}; $\approx$ 0.48 ft^3

MATH TIP

1 ft^3 = 1,728 in.^{3}

ACTIVITY 26 Continued

1–3 Activating Prior Knowledge, Quickwrite, Create Representations, Visualization, Group Presentation
As with earlier problems of this type, students should be reminded to use the formula sheet located in the back of the book. Students should share their sketches, work, and answers with the group. Continue to work with students on labeling their sketches accurately by placing dimensions where they belong in the sketch. Group presentations will help this process, allowing students to view the work of others and edit their own work.

TEACHER to TEACHER

Item 3b asks students to express the volume of the cones in cubic inches and in cubic feet. Remind students to use the relationship given in the Math Tip signal box. Debrief students' methods for performing the conversion. This will help all students understand that they should divide the volume in cubic inches by 1728 in.^{3} in order to express the volume in cubic feet.

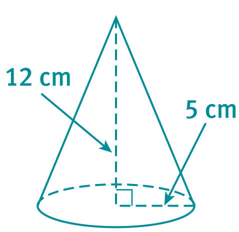
4–5 Activating Prior Knowledge, Create Representations, Visualization, Group Presentation, Debriefing In Item 4, students should consider that there are three hemispheres on the castle, and understand that this is equivalent to 1.5 spheres. In other words, students may want to find the volume of a sphere of radius 2 inches and then multiply their result by 1.5. Other solution methods are possible, and it will be worthwhile to debrief students' work so that the class is exposed to a variety of solution pathways.

Check Your Understanding

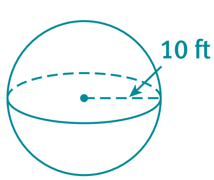
Use these items as a formative assessment to check that students can sketch a cone, sphere, or cylinder based on a verbal description and then calculate its volume. Debriefing students' responses with the class will be helpful to students who may need extra support to master these skills.

Answers

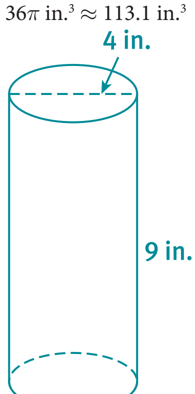
6. 100π cm$^3 \approx 314.2$ cm^3



7. $\frac{4000\pi}{3}$ ft$^3 \approx 4188.8$ ft^3



8. a. 36π in.$^3 \approx 113.1$ in.^{3}

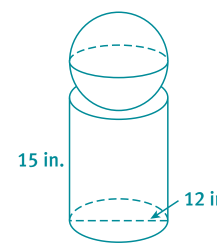


My Notes

Lesson 26-2
Volumes of Cylinders, Cones, and Spheres

4. Use the volume formula for a sphere to find how many cubic inches of sand are needed to build the three congruent decorative hemispheres on top of the wall if the radius of each hemisphere is 2 inches.
≈ 25.1 in.^{3}

5. Finally, Shayla considers the posts in front of the drawbridge.
a. Draw and label a sketch of one of the posts in front of the drawbridge if the diameter of the base of the cylinder is 12 inches and the height of the cylinder is 15 inches. The sphere on top of each post has the same radius as the cylinder.



b. Use the volume formulas for a sphere and cylinder to find how many cubic inches of sand are needed to build the two posts in front of the drawbridge. How many cubic feet are needed?
$\approx 5{,}202.5$ in.^{3}; ≈ 3.01 ft^3

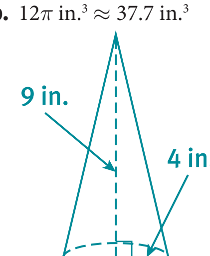
Check Your Understanding

6. Draw a cone with a height of 12 cm and a radius of 5 cm. Find the volume.

7. Draw a sphere with a radius of 10 ft. Find the volume.

8. a. Draw a cylinder with a height of 9 in. and a diameter of 4 in. Find the volume.
b. Draw a cone with a height of 9 in. and a diameter of 4 in. Find the volume.
c. How many times the volume of the cone is the volume of the cylinder?
d. State a rule for the relating the volumes of a cylinder and cone that have the same height and diameter.
e. Would your rule also apply to the volumes of a cylinder and cone that have the same height and radius?

b. 12π in.$^3 \approx 37.7$ in.^{3}

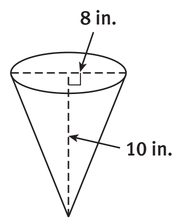


c. 3
d. If a cylinder and a cone have the same height and diameter, then the volume of the cylinder is three times the volume of the cone.
e. Yes

Lesson 26-2
Volumes of Cylinders, Cones, and Spheres

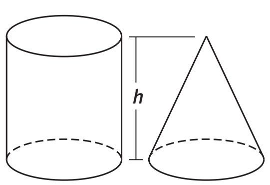
LESSON 26-2 PRACTICE

9. How is the relationship between the formula for the volume of a cone and the formula for the volume of a cylinder related to the relationship between the formula for the volume of a pyramid and the formula for the volume of a prism?
10. Find the volume of a beach ball with a radius of 12 inches. Round to the nearest tenth.
11. A glass jar has a height of 5 inches and a radius of 2.5 inches. Vanessa wants to fill the jar with beads that cost $0.12 per cubic inch. How much will it cost for her to fill the jar?
12. **a.** The figure shows the dimensions of a paper cone that will be filled with popcorn. The popcorn costs $0.02 per cubic inch. What is the cost of filling the cone with popcorn?



b. Popcorn is also sold in cylindrical tubs that have a diameter of 8in. and a height of 10 in. What is the cost of filling the cylinder with popcorn?
c. Explain how you could determine the answer to part b without using the formula for volume of a cylinder.

13. **Critique the reasoning of others.** Jason states that the volume of the cone shown below must be one-third of the volume of the cylinder since the two solids have the same height. Do you agree or disagree? Justify your response.



My Notes

ACTIVITY 26 Continued

ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 26-2 PRACTICE

9. In both cases, the volume of the figure that has one base (cone, pyramid) is one-third the volume of the figure that has two bases (cylinder, prism).
10. 7238.2 in.^{3}
11. $11.78
12. **a.** $3.35
    b. $10.05
    c. Multiply the cost of filling the cone by 3.
13. Disagree; this is only true if the two solids have the same radius.

ADAPT

Check students' work to ensure that they can calculate volumes of cylinders, cones, and spheres in a variety of mathematical and real-world settings. If students need additional practice, assign problems from the Activity Practice. Also, students will have more opportunities to work with these solids in Lesson 26-3 when they calculate volumes of composite solids.

Developing Math Language

Throughout this activity, encourage peers to support each other and confirm understanding while identifying solid figures, identifying the proper formula for volume, and working through the formulas with the correct values substituted in the formulas. Ask students to narrate, describe, and explain the volume formula of each solid figure so that they develop the names of the solid figures as basic math sight vocabulary.

Lesson 26-3

PLAN

Materials

- calculators

Pacing: 1–2 class periods

Chunking the Lesson

Example A #1–2
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity

Ask students to make a simple chart, table, or graphic organizer that summarizes the volume formulas for prisms, pyramids, cylinders, cones, and spheres. Encourage students to refer to the previous pages of this activity and/or resources in the back of the book. Take a few minutes to have students share their work. Seeing a variety of representations for this information will be especially valuable to students who are still working to internalize the formulas and math terms. In addition, students may find it helpful to refer to these charts, tables, or graphic organizers as they calculate volumes of composite solids in this lesson.

Example A Activating Prior Knowledge, Visualization, Identify a Subtask, Think-Pair-Share In this problem, students must use the given dimensions in the figure to determine any additional dimensions that they need in order to use the volume formulas. The height of the cylinder is not explicitly given in this example. Check that students understand how to interpret the given figure in order to calculate the height of the cylinder.

Developing Math Language

This lesson introduces the term *composite solid*. First, ask students to use prior knowledge and experiences to list everyday words related to the word *composite*. Some examples are *compose*, *composition*, and *composer*. Discuss how the word *compose* means "to make" and other examples related to the meaning. A composite solid is made up of simpler solid figures. As students respond to questions or discuss possible solutions to problems, monitor oral and written language production involving this new term.

My Notes

Learning Targets:

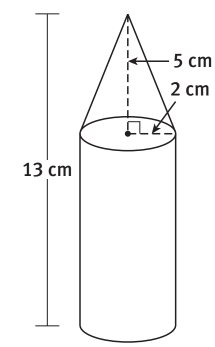
- Decompose composite solids into simpler three-dimensional figures.
- Find the volume of composite solids.

SUGGESTED LEARNING STRATEGIES: Visualization, Identify a Subtask, Think-Pair-Share, Group Presentation

A **composite solid** is a solid that consists of two or more simpler solids, such as prisms, pyramids, cylinders, cones, or hemispheres.

Example A

Find the volume of the composite solid shown below.



Step 1: Identify the solids that make up the composite figure.
The composite figure consists of a cone and a cylinder.

Step 2: Find the volume of the cone.

$V = \frac{1}{3}Bh$

$= \frac{1}{3}\pi(2)^2(5)$

$= \frac{20\pi}{3} \approx 20.9 \text{ cm}^3$

Step 3: Find the volume of the cylinder.
Note that the height of the cylinder is $13 - 5 = 8$ cm.

$V = Bh$

$= \pi(2)^2(8)$

$= 32\pi \approx 100.5 \text{ cm}^3$

Step 4: Add the volume of the cone and the cylinder.

$V = 20.9 + 100.5 = 121.4 \text{ cm}^3$

Solution: $V \approx 121.4 \text{ cm}^3$

Lesson 26-3
Volumes of Composite Solids

My Notes

Try These A

a. Sketch a composite figure consisting of two congruent square pyramids, joined at the bases, with a base edge length of 4 cm and an overall height of 12 cm.

b. Calculate the volume of the composite figure.

1. **Model with mathematics.** Shayla realizes that many parts of her castle design could be considered composite solids. Use composite solids and the calculations you made in Lessons 26-1 and 26-2 to find the total number of cubic inches of sand Shayla needs to build her castle. Show your work.

front wall:	3,456.0 in.^3
guardhouses:	216.0 in.^3
towers:	4,398.2 in.^3
turrets:	837.8 in.^3
hemispheres:	25.1 in.^3
posts:	5,202.5 in.^3
total:	14,135.6 in.^3

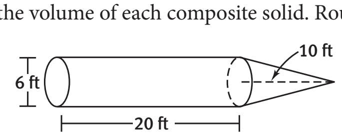
2. How many cubic feet of sand will Shayla need?

$\approx 8.2\ \text{ft}^3$

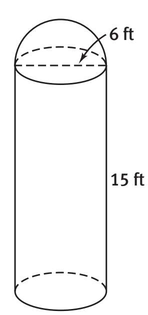
Check Your Understanding

Find the volume of each composite solid. Round to the nearest tenth.

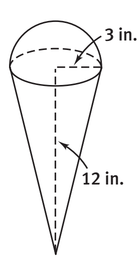
3.



4.



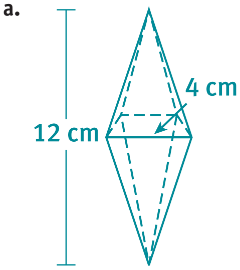
5.



ACTIVITY 26 Continued

Try These A

a.



b. 64 cm^3

1–2 Visualization, Identify a Subtask, Think-Pair-Share, Group Presentation, Debriefing Students will need to organize their information carefully in order to be sure they have included all the necessary volumes of the parts of the castle. Refer students to the castle picture in Lesson 26-1 as a visual and contextual support. While debriefing the class, discuss the organizational schemes students used. This will allow some students to benefit from the work of others and add to their knowledge of organizational systems.

Ask students to justify their reasoning and the reasonableness of their solutions. Remind students to use specific details and precise mathematical language in their justifications.

Check Your Understanding

Use these items as a formative assessment to check that students can find the volume of a composite solid. In some cases, more than one solution pathway may be possible. Take some time to debrief various solution methods with the class. Seeing multiple approaches that lead to the same correct answer is a powerful learning opportunity for students who are still working to master the material.

Answers

3. 659.7 ft^3
4. 480.7 ft^3
5. 169.6 in.^2

ASSESS

Students' answers to lesson practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 26-3 PRACTICE

6. Answers may vary. A capsule for medicine is a composite figure composed of a cylinder with a hemisphere on either end.
7. $85\frac{1}{3}$ cm^3
8. 1608.5 in.^{3}
9. **a.** 60 ft^3
   b. 3744 pounds
10. No; she will need 8 times as much sand. In any of the volume formulas, if the dimensions are all doubled, the volume increases by a factor of 8.

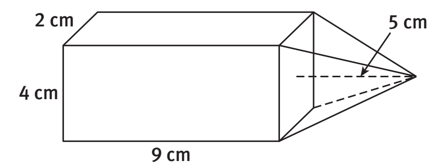
ADAPT

This Lesson Practice serves as a culmination of the activity, as it requires the use of formulas and concepts from the entire activity. Use students' work on the Lesson Practice to identify misconceptions, gaps in students' understanding, and/or areas in which they may need further practice. Assign problems from the Activity Practice, as needed, to address these issues.

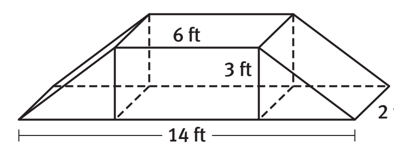
My Notes

LESSON 26-3 PRACTICE

6. Describe a composite solid you have seen in the real world. Explain how the composite figure is made up of simpler solids.
7. Find the volume of the composite solid shown below.



8. Find the volume of a composite figure comprised of two cones that are joined at the congruent circular bases, where one cone has a base radius of 8 inches and a height of 14 inches and the other cone has a height of 10 inches. Round to the nearest tenth.
9. A portable barrier that is used at construction sites is composed of three prisms, as shown below.



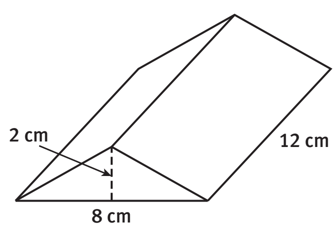
   a. What is the volume of the barrier?
   b. The barrier is made of hollow, lightweight plastic for easy transportation. Once the barrier is placed at the construction site, it is filled with water. Given that water weighs 62.4 pounds per cubic foot, what is the weight of the barrier when it is filled?
10. **Construct viable arguments.** If Shayla builds a sand castle with dimensions twice as large as the dimensions of Shelly's winning castle, will Shayla need twice as much sand? Provide an argument to justify your response.

ACTIVITY 26 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 26-1

1. Find the volume of a rectangular prism with a length of 5 inches, width of 8 inches, and height of 6 inches.
2. Find the volume of a cube with side lengths of 7.1 millimeters.
3. Find the volume of a square pyramid with a base edge length of 12 centimeters and a height of 20 centimeters.
4. Find the volume of the solid shown below.



5. A rectangular prism has a volume of 80 cubic feet. The length of the prism is 8 feet and the height of the prism is 4 feet. What is the width of the prism?
6. Jayden has a planter box in the shape of a cube. Each edge is 1.5 feet long. He fills the box with sand that weighs 100 pounds per cubic foot. Which of the following is the best estimate of the weight of the sand in the box once it is filled?
   A. 150 pounds **B.** 230 pounds
   C. 300 pounds **D.** 330 pounds
7. A cube has edges of length 6 inches. Casey calculates the surface area and the volume of the cube and states that the surface area equals the volume. Do you agree or disagree? Explain.
8. A square pyramid has a volume of 60 cubic meters. The height of the pyramid is 5 meters.
   a. What is the area of the base of the pyramid?
   b. What is the length of each edge of the square base?
9. A square pyramid has edges of length p and a height of p as well. Which expression represents the volume of the pyramid?
   A. $\frac{1}{3}p^3$ **B.** $\frac{1}{3}p^2$
   C. $\frac{1}{3}p^2 + p$ **D.** $\frac{1}{3}p + p^2$

Lesson 26-2

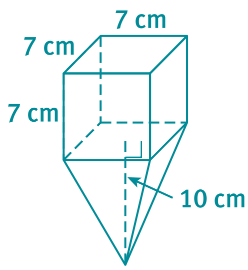
10. Find the volume of a cone with a radius of 3 inches and a height of 12 inches. Round to the nearest tenth.
11. Find the volume of a sphere with a radius of 9 centimeters. Round to the nearest tenth.
12. Find the volume of a cone having a base circumference of 36π meters and height of 12 meters. Leave your answer in terms of π.
13. What is the formula for the volume of a cone with radius r and a height of $2r$?
14. A regulation NBA basketball has a diameter of 9.4 inches. What is the volume of one of these basketballs? Round to the nearest tenth.
15. A cylinder has a volume of 18π cubic inches. The radius of the cylinder is 3 inches. What is the height of the cylinder?

ACTIVITY 26 Continued

ACTIVITY PRACTICE

1. 240 in.^{3}
2. 357.911 mm^3
3. 960 cm^3
4. 96 cm^3
5. 2.5 ft
6. D
7. Disagree; although the numerical values are the same (36), the units are different (square inches versus cubic inches).
8. **a.** 36 m^2
   b. 6 m
9. A
10. 113.1 in.^{3}
11. 3053.6 cm^3
12. 1296π m^3
13. $V = \frac{2}{3}\pi r^3$
14. 434.9 in.^{3}
15. 2 in.

16. D
17. 13.5 in.
18. C
19 **a.** 72π cm² ≈ 226.2 cm²
b. 54π cm² ≈ 169.6 cm²
20. 523.6 in.²
21. 320.7 in.²
22. Answers may vary. For the composite solid shown, $V = 506.3$ cm³.

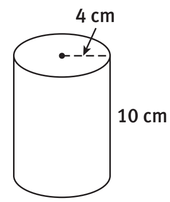


23. C
24. **a.** $r \approx 59.3$ in.; $V \approx 248.269$ in.³
b. $V \approx 248.475$ in.³
c. The difference in volumes is about two tenths of a cubic inch. The volume calculated without rounding and with the π key is more accurate.

ADDITIONAL PRACTICE

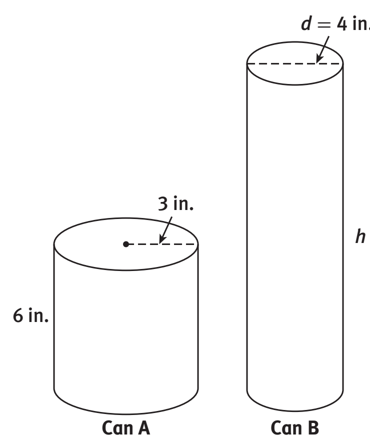
If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.

16. Which is the best estimate of the amount of soup that can fit in a soup can with the dimensions shown below?



A. 60 cm³ **B.** 125 cm³
C. 170 cm³ **D.** 500 cm³

17. You buy two cylindrical cans of juice, as shown in the figure below. Each can holds the same amount of juice. What is the height of can B?

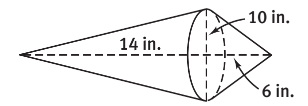


18. Which of these solids has the greatest volume?
A. a cylinder with radius 3 cm and height 3 cm
B. a cone with radius 3 cm and height 3 cm
C. a sphere with radius 3 cm
D. a cube with edges 3 cm long

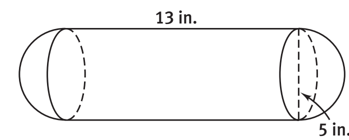
19. A cylindrical glass has a radius of 3 cm and height of 14 cm. Elena pours water into the glass to a height of 8 cm.
a. What is the volume of the water in the glass?
b. What is the volume of the empty space in the glass?

Lesson 26-3

20. Find the volume of the composite solid below. Round to the nearest tenth.



21. Find the volume of the composite solid below. Round to the nearest tenth.



22. Create a sketch of a composite solid with a total volume greater than 500 cm³. Give the volume of the figure.

23. A composite solid consists of a cube with edges of length 6 cm and a square pyramid with base edges of length 6 cm and a height of 6 cm. Which is the best estimate of the volume of the solid?
A. 100 cm³ **B.** 200 cm³
C. 300 cm³ **D.** 400 cm³

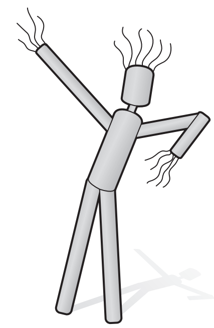
MATHEMATICAL PRACTICES
Use Appropriate Tools Strategically

24. Can rounding make a difference in your results when you calculate a volume? Consider a sphere with a radius of 3.9 inches.
a. Calculate the volume of the sphere by first finding r^3. Then round to the nearest tenth. Calculate the volume using this value of r^3 and 3.14 for π.
b. Now calculate the volume without rounding the value of r^3 and by using the π key on your calculator.
c. How do the results compare? Which value do you think is more accurate? Why?

Surface Area and Volume
AIR DANCING

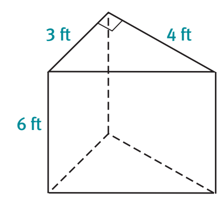
Embedded Assessment 5
Use after Activity 26

A group of students who will be attending the new Plato Middle School want to find a way to welcome the entire student body on the first day of school. After some investigation, the students decide an air dancer is a good idea and begin brainstorming ideas. The design they finally agree on has two cylindrical legs, a rectangular prism for a body, two right triangular prisms for arms, a cylindrical neck, a spherical head, and a cone-shaped hat.



Note: The drawing at the right does not necessarily represent the design that the students chose. To complete the items below, make your own drawing, showing the correct shape for each body part.

1. Sketch the air dancer design that the students chose.
2. The students must consider fans to keep a certain amount of air moving in the dancer. In order to determine the amount of air needed to inflate the air dancer, the students must calculate the volume.
   a. Find the volume of the cylindrical legs if each one is 10 feet tall and 2 feet in diameter.
   b. Find the volume of the rectangular prism to be used for the body. The dimensions are 6 feet long, 4 feet wide, and 8 feet high.
   c. Find the volume of the right triangular prisms used for arms. A diagram for one of the arms is shown to the right.
   d. Find the volume of the cylindrical neck if it is 2 feet tall and has a diameter of 1.5 feet.
   e. Find the volume of the spherical head if it has a radius of 3 feet.
   f. Find the volume of the cone-shaped hat if it has a radius of 3 feet and a height of 4 feet.
   g. What is the total volume of the air dancer?



3. a. Find the lateral area of one of the cylindrical legs using the dimensions from Item 2a.
   b. Find the lateral area of each of the right triangular prisms that are used for arms using the dimensions from Item 2c.
   c. The air dancer will be placed on a box that is a rectangular prism. The dimensions of the box are 12 feet long by 6 feet wide by 4 feet high. What is the total surface area of the box?

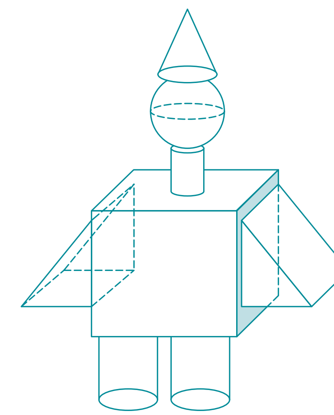
Embedded Assessment 5

Assessment Focus

- Calculate the surface area and lateral area of three-dimensional figures
- Calculate the volume of three-dimensional figures, including composite solids

Answer Key

1. Drawings may vary.



2. a. $V = 20\pi \approx 62.83$ ft^3
   b. $V = 192$ ft^3
   c. $V = 72$ ft^3
   d. $V = 1.125\pi \approx 3.53$ ft^3
   e. $V = 36\pi \approx 113.10$ ft^3
   f. $V = 12\pi \approx 37.70$ ft^3
   g. $V \approx 481.16$ ft^3
3. a. $LA = 20\pi \approx 62.83$ ft^2
   b. $LA = 72$ ft^2
   c. $SA = 288$ ft^2

Common Core State Standards for Embedded Assessment 5

8.G.C.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Teacher to Teacher

You may wish to read through the scoring guide with students and discuss the differences in the expectations at each level. Check that students understand the terms used.

Embedded Assessment 5
Use after Activity 26

Surface Area and Volume
AIR DANCING

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2a-g, 3a-c)	• Accurately and efficiently finding the surface area and volume of three-dimensional figures.	• Finding the surface area and volume of three-dimensional figures with few, if any, errors.	• Difficulty finding the surface area and volume of three-dimensional figures.	• No understanding of finding the surface area and volume of three-dimensional figures.
Problem Solving (Items 2a-g, 3a-c)	• An appropriate and efficient strategy that results in a correct answer.	• A strategy that may include unnecessary steps but results in a correct answer.	• A strategy that results in some incorrect answers.	• No clear strategy when solving problems.
Mathematical Modeling / Representations (Item 1)	• Precisely modeling a problem situation with an accurate diagram.	• Drawing a reasonably accurate diagram to model a problem situation.	• Difficulty drawing a diagram to model a problem situation.	• Drawing an incorrect diagram to model a problem situation.
Reasoning and Communication (Items 3a-c)	• Correctly understanding the difference between total surface area and lateral surface area.	• Distinguishing between total surface area and lateral surface area.	• Confusion in distinguishing between total surface area and lateral surface area.	• No understanding of the difference between total surface area and lateral surface area.